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New Features

Since the first typewritten issue of May, 1956, these TRANSACTIONS have undergone several changes. "The Issue in Brief" section was added along with an annual membership directory, and the format was changed considerably when typesetting was introduced.

In this issue there are other new features: author's biographies and photographs are included as well as those of the PGAC administrative committee. It is intended that authors will be introduced to the readers in each future issue and that new administrative committee members will be presented each year.

Another new feature which we hope will become popular is the correspondence section. This is new in the sense that, unlike the featured papers, these letters will include short, informally written articles describing current control problems or discussing papers printed in past issues. Correspondence will not be reviewed or returned to the writer for any changes, and no biographies and/or photographs will be included in the Contributors' Section. This is the procedure followed in the PROCEEDINGS OF THE IRE.

In the future, other features will be added. For example, automatic control is achieving formal national and international status with the formation and growth of the AACC (American Automatic Control Council) and IFAC (International Federation of Automatic Control), and the PGAC, a participant of AACC, will feature articles on the activities of these groups and the contributions that they may make to the field of automatic control. In this issue, plans for the First International Congress of IFAC at Moscow in 1960 are announced.

Another feature reserved for future issues will be the inclusion of abstracts from other journals, national and foreign. It will not be possible to do this immediately, however, because procedures for providing regular abstracts have not yet been established, largely because of the expense involved. However, translating agencies are becoming more prevalent, and organizations such as IFAC are expected to make communication between various countries less difficult.

Because of the relatively high number of rejections by the PGAC reviewers, the accumulation of papers has not been large enough to provide more than two regular issues a year. However, this number may be increased to three and eventually to four. We hope that, with the addition of these new features, these TRANSACTIONS will grow and have more value for members of the PGAC. Comments and suggestions are always welcome.

—*The Editor*

The Issue in Brief

As in the past, a variety of automatic control subjects are considered in this issue. This time, however, three of the papers concern practical equipment and applications where past issues have featured papers of a more theoretical nature.

Feedback System Testing, **Charles F. White**—Page 79

Direct methods of testing control systems, especially in production, to correlate actual performance with theory are particularly valuable in improving system reliability and understanding. In this paper an analog method of servo system performance testing applicable to experimental analysis and system development and to go—no-go production and maintenance testing has been devised. Usually, sinusoidal testing methods are most effective with the servo system loop open, and transient testing (single-event time-domain input) is most effective with the loop closed. The analog method described, as distinguished from these transient and sinusoidal methods, uses *time*-domain signals (a step-function signal generator input is found suitable) to obtain *frequency*-domain parameters in a method effectively making an *open-loop test of a closed-loop servo system.*

Single-integrator, angle tracking, and range tracking servo systems have been analyzed, and the testing of a high-performance automatic range tracking system for radar moon echoes is described.

Considerations in Phase Shifting, **M. G. Rekoff, Jr.**—Page 89

Two phase servo motors are often operated from a single phase source using various phase shifting methods. The different possibilities of obtaining quadrature voltages at the terminals of an ac servomotor with static impedance elements, connected to the reference winding, are discussed together with the limiting restrictions for each. Formulas for specifying elements of the phase shifting impedance are given in terms of motor constants for several of the more common impedance configurations.

Analysis of Gyro Orientation, **Arthur Mayer**—Page 93

Although gyros have been used for a number of years in military equipment to provide space references, their use is becoming greater in the missile age. It is more important than ever for engineers to understand the dynamic relationships that occur between control loops due to various gyro orientations. Often, the frequency response of a gyro-stabilized platform is limited by the gyros. For the most effective suppression of instability with a minimum sacrifice in bandwidth, it is necessary to orient single degree-of-freedom gyros in a certain way. This paper discusses the mathematical relationships of different gyro orientations, and it indicates the mounting usually preferred.

A Survey of Adaptive Control Systems, **J. A. Aseltine, A. R. Mancini, and C. W. Sarture**—Page 102

There has been considerable work in recent years on self-optimizing, self-adjusting, or adaptive control systems. Much of the literature is overlapping; there has been little attempt made to classify or explain the difference between the various systems and the methods used to achieve the optimum state.

In this paper, the various criteria upon which self-optimizing systems have been based are reviewed, and the operation of each system is discussed. A new system which is self-optimizing with respect to a measure of impulse response is described, and experimental results are presented.

Stability of Forced Oscillations in Nonlinear Feedback Systems, **Ze'ev Bonenn**—Page 109

It has been known for a considerable time that nonlinear systems may exhibit multivalued response under external periodic excitation. The stability of these forced oscillations has been extensively investigated for second order systems and a general criterion for their stability has been found. Stability of higher order systems was investigated by means of the incremental describing function. This function must be calculated for every case of interest.

In this paper the general criterion formerly derived for second order systems is extended to higher order systems. Thus it is not necessary to make a special stability investigation in every case.

A Selective Bibliography on Sampled Data Systems, **Peter R. Stromer**—Page 112

In the March issue of these TRANSACTIONS, a rather extensive "Bibliography of Sampled-Data Control Systems and Z-Transform Applications" by Freeman and Lowenschuss was published. Before the issue had been distributed, however, another bibliography, compiled by P. R. Stromer, was received. Although it duplicates many of the references in the earlier paper, it is included in this issue because, with a brief description of each reference, it provides a fine supplement to the earlier bibliography.

No attempt has been made to cover material before 1955 except where particular references have been cited by various authors repeatedly, indicating "classic" references in this comparatively new area of feedback control system literature. Items are listed alphabetically by author.

Contributors—Page 114

Authors' biographies and photographs are included for the first time. Biographies of the members of the PGAC Administrative Committee are also presented in this issue.

Correspondence—Page 122

A letter concerning the proposed standard terminology presented in the March issue is published, along with a discourse on the relationship between operations research and its potential when used in conjunction with computers.

Feedback System Testing*

CHARLES F. WHITE†

Summary—An analog method of servo system performance testing applicable to experimental analysis and system development and to go—no-go production and maintenance testing has been devised. For a servo system with a forward transfer function G_{eo} and a feedback transfer function G_{ou} the loop actuating signal equals the loop input signal divided by $1+G_{eo}G_{ou}$. The loop actuating signal becomes exactly equal to a test signal if the test signal is multiplied by the factor $1+G_{eo}G_{ou}$ before application to the servo system. The difference between the loop actuating signal of the closed-loop servo system and the arbitrary test signal is continuously zero for the system exactly meeting the specification $G_{eo}G_{ou}$ function simulated in the signal generator. Sinusoidal testing methods are most effective with the servo system loop open. Transient testing (single-event time-domain input) is usually performed on servo systems with the loop closed. The analog method described, as distinguished from transient and sinusoidal methods, uses *time*-domain signals (a step-function signal generator input is found suitable) to obtain *frequency*-domain parameters in a method effectively making an *open-loop test of a closed-loop servo system.*

The $1+G_{eo}G_{ou}$ signal generator is conveniently realized using an analog computer of the operational amplifier type. An alternative mechanization of the signal generator uses only passive elements. Nonlinear as well as linear servo systems may be tested by the method.

Single-integrator, angle tracking, and range-tracking servo systems have been analyzed. The range-tracking transfer function was employed in an analog computer experiment. Recordings of both the loop actuating signal and the difference between the loop actuating signal and the signal generator input were made for 0, ±6 db, and ±12 db departures from correct gain and for 0, ±1, and ±2 octave departures from correct corner frequency. A study of these recordings revealed potentialities for control of adaptive servo systems.

Introduction

SERVO system testing plays an important role in the evaluation of the performance of modern systems. The usual closed-loop tests are insensitive indicators of any departure from the specified open-loop transfer function. There is a need for a method of effectively making an open-loop test of a closed-loop servo system.

The paper presents an analog method of servo system performance testing which is applicable to experimental analysis and system development and to go—no-go production and maintenance testing. The signal generator used employs an analog of the servo loop transfer function. The analog may take the form of several operational amplifiers of the analog computer type or it may be formed of entirely passive elements. No restriction to linear servo systems is involved. The same nonlinearity present in the servo system being tested may be employed in the signal generator.

The paper contains a detailed analysis of a single-integrator servo system tested by the analog method. An analysis is given of the nature of the servo system input signal using the described testing method for servo transfer functions suited to automatic angle-tracking and to automatic range-tracking systems.

Experimental results are given for a servo system with a transfer function similar to that used in airborne radar automatic-tracking range units. Variations in the test monitor output are explored as a function of loop transfer function departures from the specification value.

Importance of Closed-Loop Testing

Performance evaluation of electronic equipment is accomplished during production by unit tests, subsystem tests, and system tests. Similar performance tests, usually in the reverse order, are required during field maintenance. Feedback loops requiring performance testing are found in each of these categories in modern electronic systems. Accordingly, any possible improvement in servo system testing becomes important.

As an example of a typical electronic system, consider an airborne armament control system employing radar. The radar antenna contains azimuth and elevation servo drives with modes of operation including a) programmed control for search, b) manual control for final target acquisition, and c) automatic control during tracking. Because of the mounting, stabilization loops are required to compensate for the aircraft motion. During the search phase, signals may be added to the programmed control, and during automatic tracking, aircraft angular velocity feedback may become a part of the control loop. The radar receiver contains feedback for automatic frequency control and for automatic gain control. The radar range unit involves feedback systems a) in the search modes, and b) in the tracking mode. Some range units have both wide bandwidth and narrow bandwidth operation in tracking. Computers for solution of the kinematic and ballistic problems contain servo systems in a) lead angle computers, and b) flight data computers. These computer servo systems, in general, convert electrical signals to shaft position and vice versa. Servo loops also appear in the regulated power supplies of the system.

The example clearly supports the assertion that servo system testing plays an important role in the evaluation of the performance of modern electronic systems. The efficiency and rapidity of the method used for the performance testing of closed-loop servo systems has an important relationship to the total time required for system performance tests, since a majority of the components of a given system are parts within servo loops.

A wide variation in the characteristics of a servo system causes relatively small change in the results obtained by the usual closed-loop transient or frequency

* Manuscript received by the PGAC, January 27, 1958.
† Naval Res. Lab., Washington, D. C.

response evaluation. The more sensitive approach of open-loop testing is, unfortunately, unavailable, since the opening of the loops in completed equipment is ordinarily highly impracticable. In many practical engineering situations open-loop testing is found difficult, if not impossible. Some systems, *e.g.*, steam turbine regulating systems, are inherently unstable until the loop is closed. In other systems mechanical unbalances require closed-loop operation to maintain a given position. In these cases, interpretation of open-loop sinusoidal testing results becomes difficult. The wide range of open-loop gain over the frequency range requiring test leads to severe measurement problems. Even though open-loop measurements can be converted to equivalent closed-loop measurements though analytic means, a direct measurement of the performance of the unaltered finished product is highly preferred. Hence there is a need for a method effectively making an *open-loop test of a closed-loop servo system.*

For production and maintenance purposes, the test method should be automatically referenced to design performance specifications, since prior to performance evaluation or testing, the adequacy of design and the desired dynamic performance characteristics of the system ordinarily have been established through operational requirements, technical specifications, engineering design, evaluation studies, and mechanization studies. One object of testing, as considered in the present report is to reaffirm the *actual* dynamic system performance characteristics exhibited by particular groups of standard production items assembled into complete systems. Such performance tests are required as final steps in production and, later, as periodic maintenance checks throughout the life of the equipment in the field. Final proof of performance of aircraft and missile system aerodynamic control characteristics can be obtained only in the actual closed-loop environment. Efficient closed-loop system testing can result in great savings in these fields.

Basic Testing Concept

As previously mentioned, open-loop testing of servo system performance is, relative to the usual closed-loop tests, a much more sensitive method. To understand what happens when the loop is opened, consider Fig. 1. The immediately evident difference is that the loop actuating signal of the open-loop system is exactly equivalent to the input instead of being the result of a comparison between the input and a function of the output as in the closed-loop system. Accordingly, to force the closed-loop system performance to be equivalent in all respects to the open-loop system performance, set

$$X_{eCL} = X_{eOL}. \tag{1}$$

As indicated in Fig. 1, the loop actuating signal of the closed-loop system is

$$X_{eCL} = \frac{X_{iCL}}{1 + G_{eo}G_{ou}} \tag{2}$$

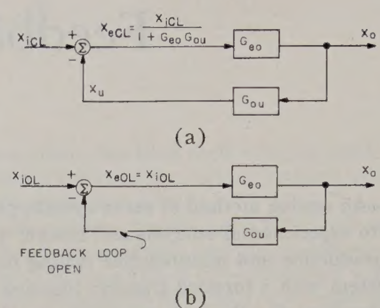

Fig. 1—Comparison of (a) closed-loop and (b) open-loop servo systems.

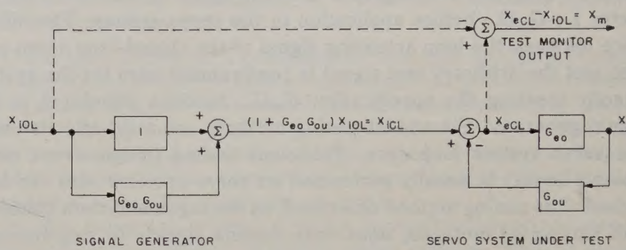

Fig. 2—Basic concepts of an analog method for servo system performance testing.

while the open-loop system actuating signal is

$$X_{eOL} = X_{iOL}. \tag{3}$$

Substituting the equivalent expressions from (2) and (3) into (1), we obtain the requirement

$$\frac{X_{iCL}}{1 + G_{eo}G_{ou}} = X_{iOL}$$

or

$$X_{iCL} = (1 + G_{eo}G_{ou})X_{iOL}. \tag{4}$$

Eq. (4) may be taken as the defining equation of a signal generator for servo system testing. Application of an input X_{iCL} to the closed-loop servo system of the form specified by (4) causes an error X_{eCL} identical to that obtained with X_{iOL} (unmodified by the $1 + G_{eo}G_{ou}$ factor) applied to the open loop. Further, the closed-loop system actuating signal and the signal generator input become identical. That is,

$$X_{eCL} - X_{iOL} = 0. \tag{5}$$

A physical analog of (4) is shown in Fig. 2. In practice, the loop actuating signal X_{eCL} may be displayed on an oscilloscope or recorded. Alternatively, (5) may be employed, as shown by the dashed lines of Fig. 2, to use a continuous zero output as an indication of performance exactly in accordance with specifications.

Any departure of the loop gain product $G_{eo}G_{ou}$ of the servo system under test from the specification value employed in the signal generator produces corresponding changes in the error signal. In applying the analog test method to go—no-go tests, acceptable limits of departure may be experimentally or analytically determined. In general, the test duration need not extend beyond 5 to 10 system time constants (reciprocal radian bandwidths).

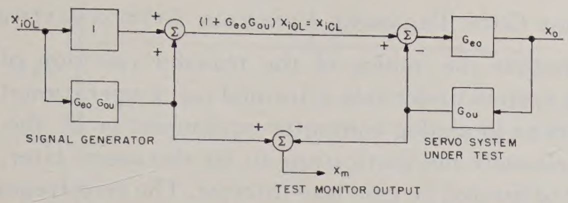

Fig. 3—Method providing a test monitor output identical to that of Fig. 2 with reduced measurement difficulty.

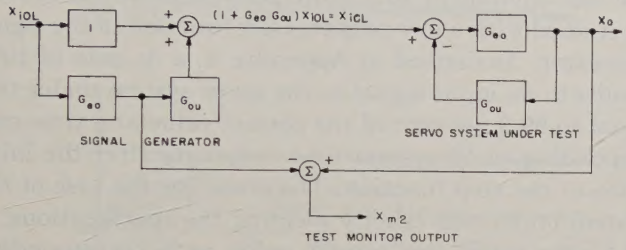

Fig. 4—Recommended monitoring method for cases where simulation of G_{ou} offers no particular difficulty.

Note that nothing has been specified regarding the nature of the signal generator input X_{iOL}. No restriction exists other than that of using a signal amplitude within the linear range of the servo system error detector. A simple step-function appears to be an easily produced signal which is well suited to the application.

In application of the analog method to analysis and development of servo systems some important aspects should be mentioned. The test method compares a servo system transfer function and a simulation of that transfer function used in formation of a signal generator. The test monitor output shows both the magnitude and sense of any difference between the transfer function gains. Nonlinear servo systems represent no limitation, since the same nonlinearities may be introduced in the signal generator analog. Progressive improvement of the analog from the first linear approximation to inclusion of known nonlinearities permits discovery of any residual unknowns.

The measurement method depicted in Fig. 2 represents a direct application of the ideas embodied in the derivation. The necessity for access to the servo system error signal imposes limitations in testing existing equipment and involves practical measurement problems. A comparison of two low-level high-impedance signals is required. Fig. 3 shows an alternate method involving only high-level measurement with an analytically identical test monitor output X_m. While the test monitoring method of Fig. 3 is superior to that of Fig. 2 for practical measurement reasons, the method of Fig. 2 was the method actually employed in the systems investigated during the development of the basic method.

Wherever the nature of the servo system feedback transfer function introduces no simulation difficulties in a separate realization, the monitoring arrangement of Fig. 4 is recommended because a) the required access to the servo system is to the normally available input and output terminals only, b) only high-level signal comparison is required in the test monitoring, and c) for systems where G_{ou} is not equal to unity the test monitor output X_{m2} shows greater sensitivity to detection of variations in G_{eo} and in G_{ou} than present in the X_m of Figs. 2 and 3.

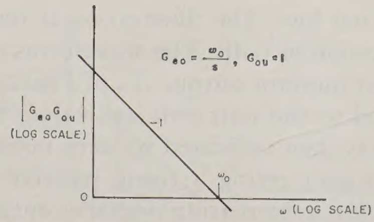

Fig. 5—Simple servo system transfer function.

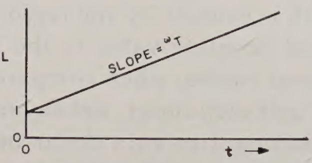

Fig. 6—Test signal for servo system of Fig. 5.

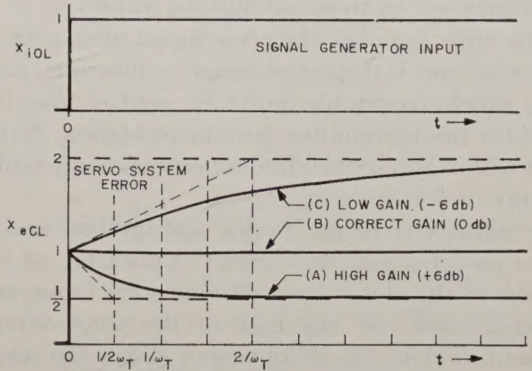

Fig. 7—Single-integrator servo system loop actuating signal for step-function input to signal generator and servo system input shown in Fig. 6.

Application to Simple Servo System

Analysis of a simple servo system tested by the method outlined in the previous paragraphs verifies the sensitivity of the procedure and its suitability for go—no-go testing. In Fig. 5, a single-integrator or Type 1 servo system transfer function is depicted on a log-magnitude log-frequency plot. The simulation in the test equipment should be made using exactly the same function and a value of ω_0 equal to the specification value. By well-known methods of derivation the form of the servo system test signal for a step-function signal generator input was found to be that shown in Fig. 6 in which the subscript T refers to "TEST" equipment characteristics. In using the method as a go—no-go test, a no-go answer can be unquestionably related to non-specification servo system performance if the signal generator output is checked and found to correspond with previous recordings. Fig. 7 shows the appearance of the servo system loop actuating signal for the three possible conditions of gain and bandwidth: a) too high, b) cor-

rect, and c) too low. The illustration is for departures from specification of 6 db. The waveforms of Fig. 7 apply to the test monitor output X_m of Fig. 2, if the reference is shifted to the unit ordinate value. Specification performance is then indicated by zero output with low servo system gain giving a rising positive output and high gain giving a decreasing negative output. The initial rate of change is slow for low gain, and the equivalent too-narrow bandwidth is evident. Likewise, the too-wide bandwidth is evident by the rapid rate of decrease toward the final negative value in the high-gain situation. Error signal values, when compared with the amplitude of the unit step input, are asymptotic to values inversely proportional to gain departures from specification gain. Readings at a time 5 to 10 system time constants after application of the step-function are good approximations to these asymptotic values.

If the error signal or the error signal minus the signal generator input is displayed on an oscilloscope, an overlay on which acceptable limits are scribed may be employed for production line go—no-go testing. Any indication within the scribed limits becomes acceptable performance and vice versa.

The sensitivity of the analog test method under discussion as compared with direct application of a step-function to the closed-loop servo system input may be examined easily for the case of the single-integrator transfer function. Fig. 8 (opposite) shows the results of the usual transient testing.

With direct application of a step-function, the final values for the error or for the output are common asymptotes, as seen in Fig. 8. That is, relatively little distinction between various gain levels is observable at, say, 10 system time constants after initiation of the input. In the case of the simple system under discussion, the initial slopes are directly related to loop gain. Unfortunately, the region of greatest accuracy is also a region of small differences between responses. In practice, the response to be used as the reference requires the use of considerable care in establishing and maintaining the required accuracy. These difficulties obviously are not present in the null indication of the analog method. In the analog method, the asymptotic values of the error were inversely related to departures of loop gain from the specification value. As a practical matter, these final values are very satisfactory as compared to transient indications.

As the complexity of the servo system transfer function increases, the transient analysis difficulties described above increase rapidly, and little possibility of a direct determination of system transfer function remains. In the analog method, a completely unknown system of complicated nature would offer great difficulty. However, one or two unknown corner frequencies in a complicated function can be determined without excessive difficulty.

Loop Gain Transfer Function Approximations

Whether the analog of the transfer function of the servo system under test is formed using operational amplifiers as in analog computer equipment or by the passive element mechanization to be discussed later, the effect of limited dc gain is of interest. The zero-frequency gain of the single-integrator system previously studied was theoretically infinite. The effect of a limitation in the test equipment to a finite zero-frequency gain may be studied with a low-pass transfer function in the signal generator. As derived in Appendix A, a dc gain of 1000 results in an input signal to the servo system under test equal to 99.5 per cent of the correct value at a time corresponding to 10 system time constants after the initiation of the step function. The error, for the case of the system under test exactly meeting the specifications, is 99.1 per cent of the correct value at a corresponding time.

It is concluded that a zero-frequency dc gain of 1000 is adequate for the usual engineering purposes. In general, electronic servo systems (as distinguished from servo-mechanisms) are inherently of limited gain at zero frequency. In such cases, the possible deficiencies of the analog would be even less important.

Alternative Mechanizations of Signal Generator

The signal generator concepts of Figs. 2, 3, and 4 require in the $G_{eo}G_{ou}$ portion a variation of transmission not only as a function of frequency but also of gain to properly analog the loop gain specified for the servo system under test. While the contemplated mechanization is perhaps desirable for airborne testing as in systems for in-flight built-in test, in ground applications as a production-line go—no-go test method the restrictions on size and weight are not present and the effective capacitance multiplication through the use of electronic gain (Miller integrators) may be replaced with passive networks. Fig. 9 shows a passive network signal generator mechanization in which K_T is to be made equal to the dc gain of the servo system under test.[1] As stated in the previous section, a dc gain of 1000 was a satisfactory approximation to the theoretical infinite dc gain of a single-integrator servo system.

The voltage for the step-function signal generator may be derived from a source incorporated in the signal generator. However, the dc supply of the system under test may be used. For a typical airborne angle-tracking servo system, the gain at dc is ordinarily less than 1000. With a 300-volt dc supply and an error-signal step of 0.1 volt, a value for K_T of 3000 is available.

In Fig. 9, the $G_{eo}G_{ou}/K_T$ portion may be obtained using entirely passive networks exhibiting the variation in transmission specified for the servo system under test.

[1] This idea for mechanization of the basic concept is due to L. F. Gilchrist of the Equipment Res. Branch, Naval Res. Lab.

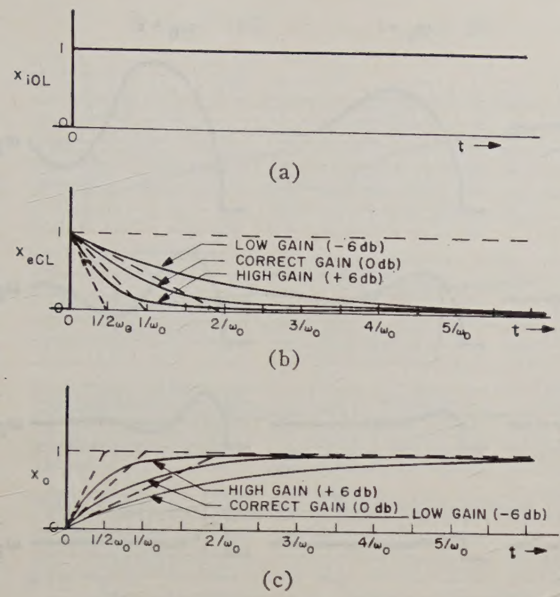

Fig. 8—Step function input applied directly to a closed-loop servo system (a) and the corresponding error (b) and output (c) for departures from the specification loop gain of −6 db, 0 db, and +6 db.

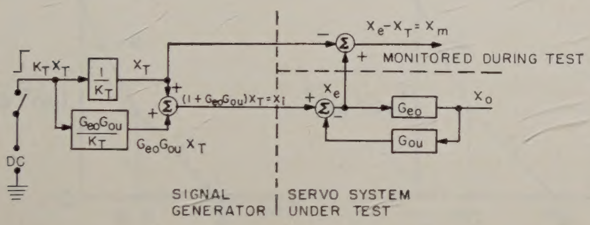

Fig. 9—Passive network mechanization of the servo system performance testing method providing zero output at the monitored output when the servo system exactly meets specifications.

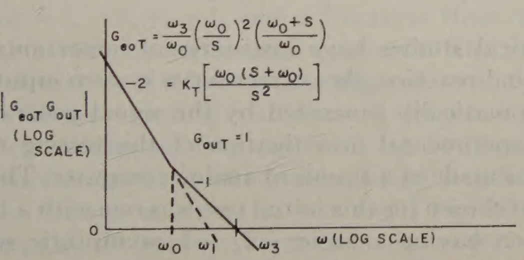

Fig. 10—Range-tracking servo system asymptotic transfer function as used in the signal generator.

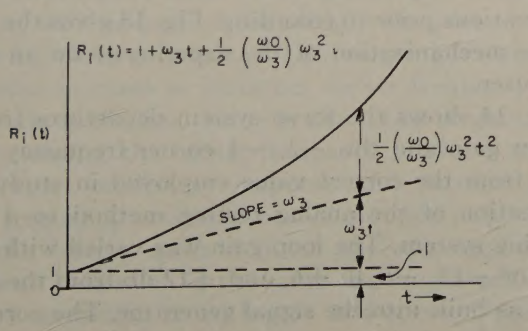

Fig. 11—Test signal for a range tracking servo system.

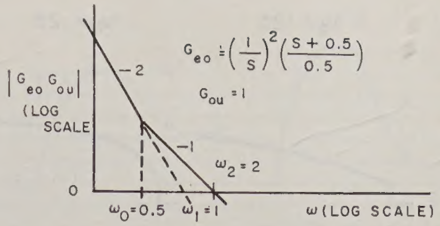

Fig. 12—Servo system transfer function used in the analog computer experiment.

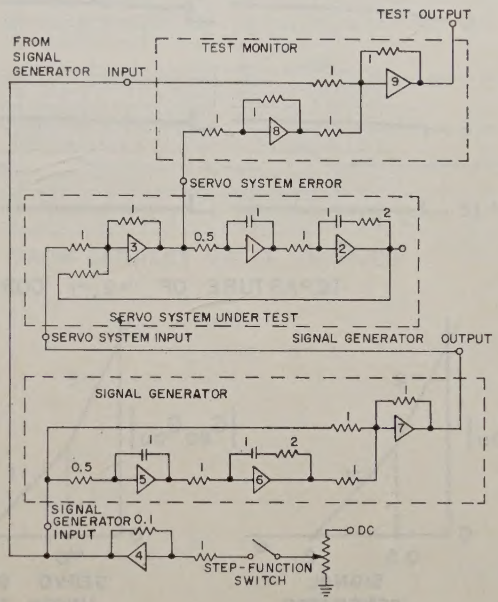

Fig. 13—Experiment testing a range-tracking servo system using the analog method. Units used are megohms and microfarads.

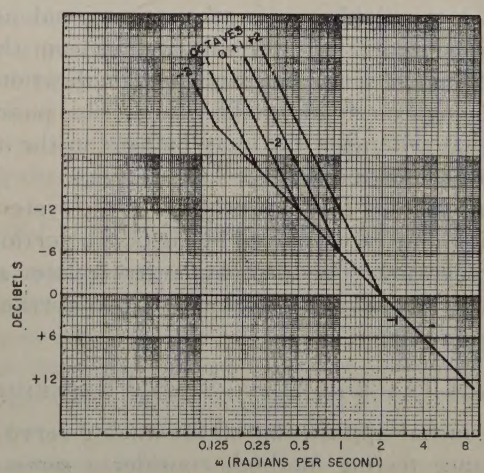

Fig. 14—Servo system departures from correct gain (decibels) and departures of −2, −1, corner frequency from correct value (octaves) investigated using the analog testing method. Results are shown in Figs. 15 and 16.

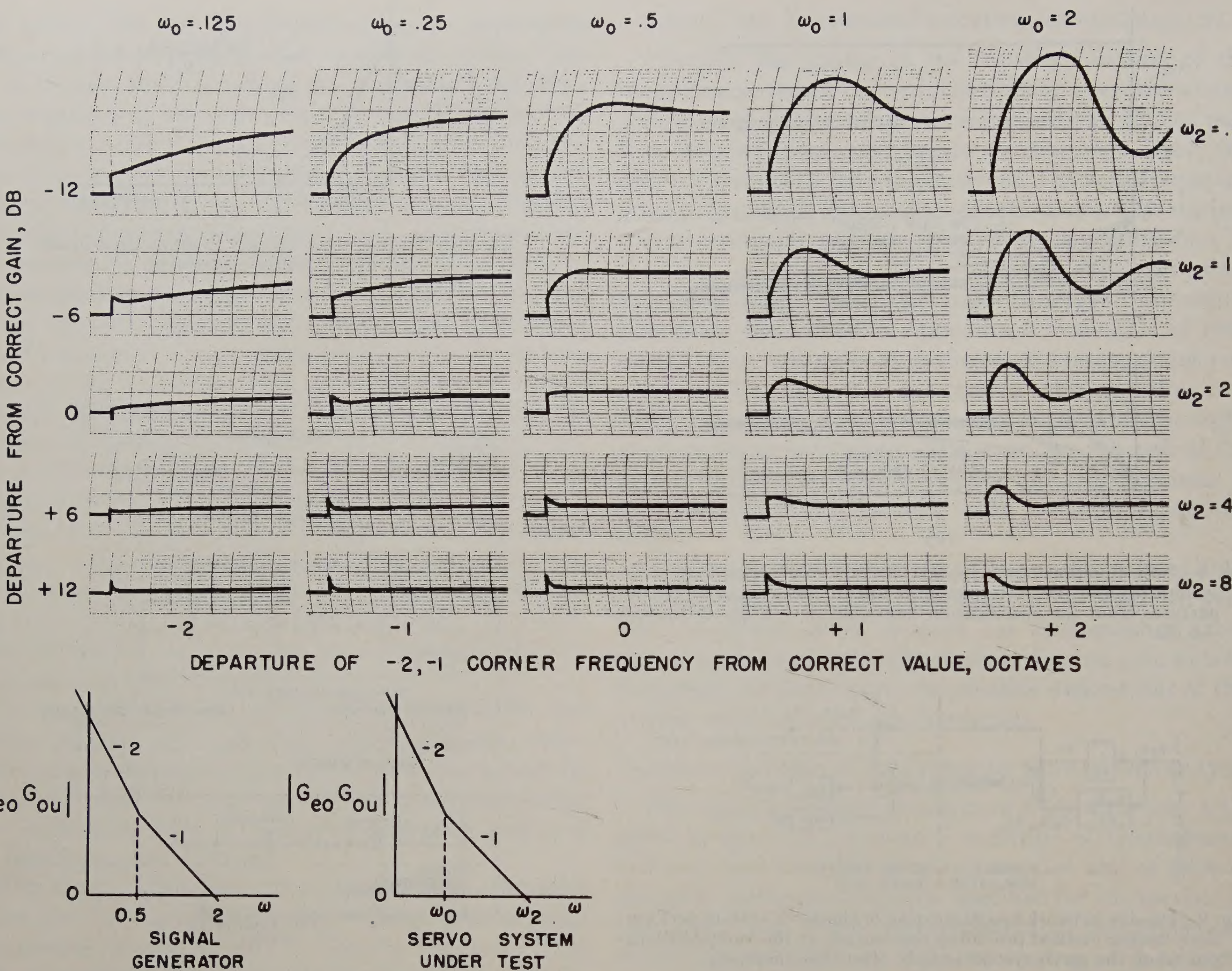

Fig. 15—Loop actuating signal in the range-tracking servo system experiment.

The other portions of the signal generator may be formed of resistance networks. Since the multiplication of X_T by K_T is later cancelled by a division of unity by K_T, X_T is available, as in the operational amplifier mechanization of Fig. 2 for subtraction from the servo error and use of a zero output as an indication of the system being exactly on specifications. This passive network signal generator idea may be used in the arrangements of Figs. 3 and 4.

In some cases, a tested prototype of the system under production may be employed in the $G_{eo}G_{ou}$ portion of the signal generator. The prototype servo loop may be opened and connected to form the basic portion of the signal generator.

Range-Tracking Servo System Experiment

As an initial application of the analog servo system performance testing method, consider a servo system transfer function suitable for automatic range tracking. The basic portions of a typical radar range-tracking servo system transfer function are shown in asymptotic fashion in Fig. 10. Analysis shows that the input to such a servo system takes the form shown in Fig. 11. Such analytical studies have instructional importance only. In actual practice, the correct servo system input signal is automatically generated by the signal generator. In this experimental investigation of the testing method use was made of a standard analog computer. The servo system chosen for this initial test was one with a transfer function having a basic -2, -1 asymptotic segment combination as shown in Fig. 12. The constants shown were chosen for a time scaling suited to display of the test results on a vacuum-tube voltmeter for initial visual observations prior to recording. Fig. 13 gives the details of the mechanization of the experiment on an analog computer.

Fig. 14 shows the servo system departures from the correct gain and the -2, -1 corner frequency departures from the correct value employed in studying an application of the analog testing method to a range-tracking system. The loop gain was varied with departures of -12, -6, 0, $+6$, and $+12$ db from the correct value as built into the signal generator. The corner frequency break between the low-frequency double-integrator asymptotic slope and the high-frequency single-integrator asymptotic slope was varied over the

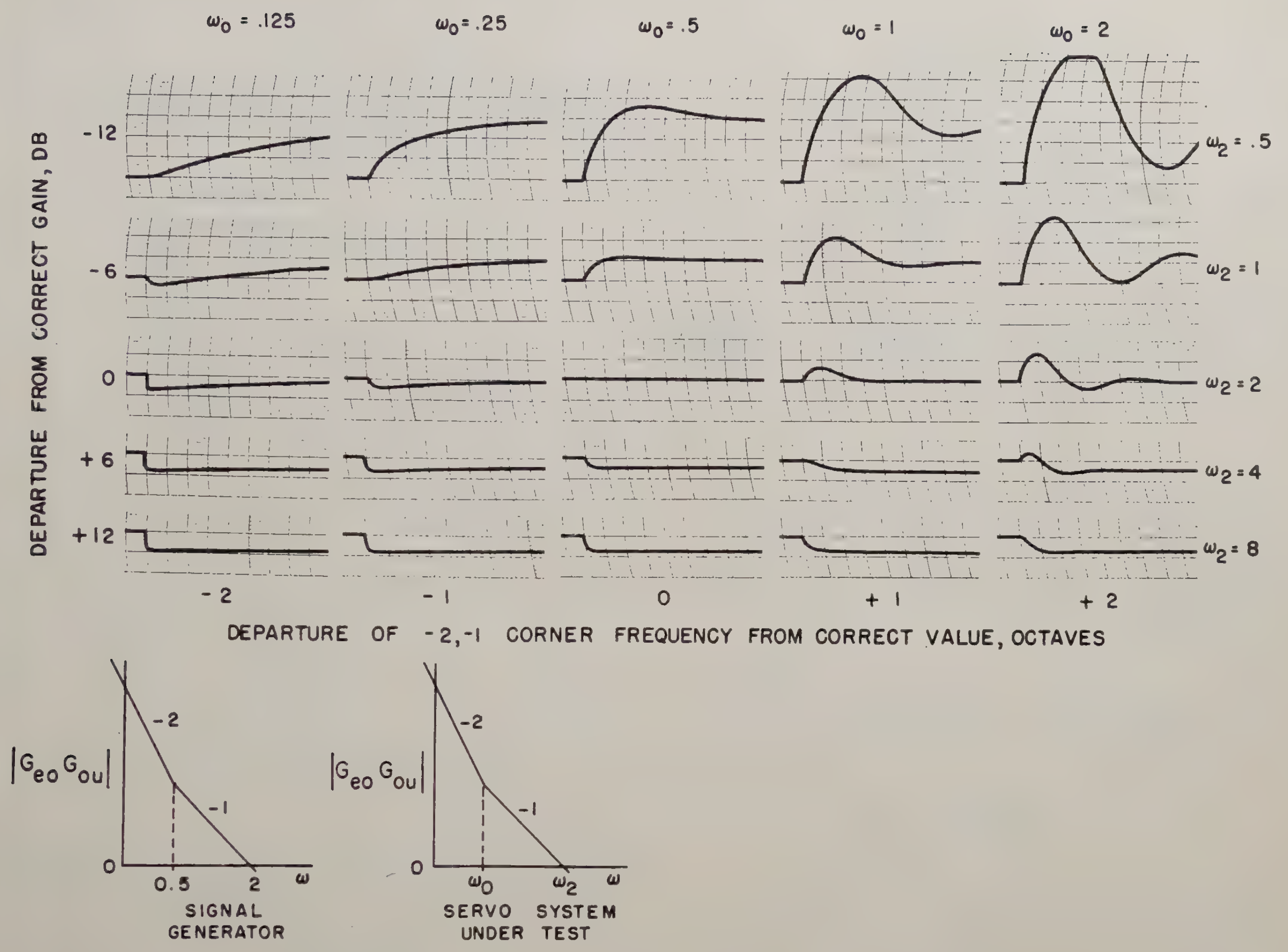

Fig. 16—Test monitor output in the range-tracking servo system experiment.

range of -2, -1, 0, $+1$, and $+2$ octaves from the correct value built into the signal generator. The result is a 5 by 5 matrix of 25 cases investigated with the 0, 0 center case representing conditions of the servo system under test exactly meeting specifications. In all cases the same positive (upward on the recordings) step-function input to the signal generator was used. Fig. 15 shows recordings of the servo system error signal for a step-function input to the signal generator. Note that the initial step appears at the beginning of all error signal traces. The initial error is seen to grow, stay constant, or decrease, with respect to the initial value, for low, correct, or high loop gain values. The foregoing is true of the *final value* in cases of incorrect corner frequency. The magnitude of the final value is a function of gain. Note that increasing the corner frequency produces a tendency for an oscillatory response and decreasing the corner frequency produces an overdamped response. In using the testing method in an analyzer application, there is an indication of the sense and magnitude of the required correction to both gain and corner frequency.

Fig. 16 shows recordings of the test monitor output, *i.e.*, the difference obtained in subtracting the signal generator input from the error signal. Note that the indications for a positive step-function input to the signal generator are positive-going for low gain, zero for correct gain, and negative-going for high gain when the corner frequency is correct. This is also true of the *final values* when the corner frequency is incorrect. In general, there is a positive-going tendency for corner frequency high and a negative-going tendency for corner frequency low. The advantages of null methods are exploited since the indication for the servo system under test exactly meeting the specifications is a *continuous zero output.*

Passive-Element Signal Generator Experiment

The alternative mechanization of the signal generator discussed in connection with Fig. 9 envisions an elimination of any requirement for gain in the signal generator. The loop transfer function variation in transmission as a function of frequency is obtained by an entirely passive network. An application to the range-tracking servo system experiment was made.

The variation of transmission with frequency of the transfer function of Fig. 12 may be approximated by a two-section low-pass resistance-capacitance network of the form shown in Fig. 17(a). As analyzed in Appendix B, the network constants employed in Fig. 17(b) result

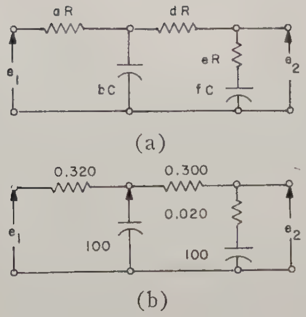

Fig. 17—Two-section low-pass resistance-capacitance network approximation to the variation in transmission as a function of frequency that is specified by Fig. 12. The units are megohms and microfarads.

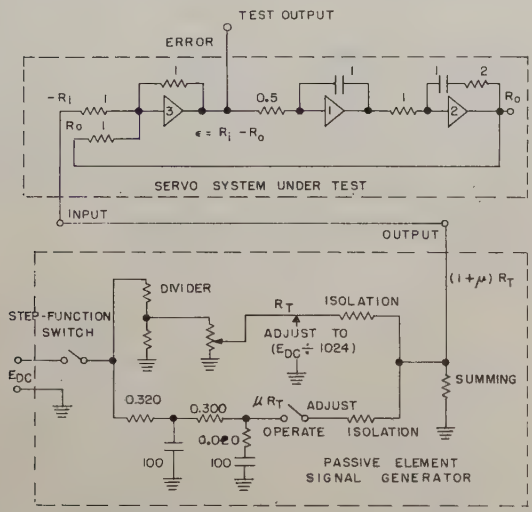

Fig. 18—Passive-element signal generator as an alternative mechanization for testing the range-tracking servo system of Fig. 13.

in a fraction equal to 1/1024 for the $1/K_T$ of Fig. 9. As determined in the study of loop gain transfer function approximations (Appendix A) , a gain of 1000 is an entirely satisfactory approximation to the theoretical infinite gain shown in Fig. 12.

The complete circuit of the passive-element signal generator used in a rerun of the range-tracking servo system experiment is shown in Fig. 18. In the arrangement of Fig. 13, an electronic error detector was employed to measure the difference between the error signal of the servo system under test and the signal generator input. To adhere to an all-passive tester idea, direct observation of the error signal itself was used here. With the otherwise completely arbitrary signal generator input restricted to a step-function, the advantages of the over-all procedure are retained without use of the null measurement method. The input dc is assumed to be available from the servo system under test. A standard laboratory vacuum-tube voltmeter may be used as the test output indicator.

Essentially identical results (within the recording sensitivity used previously) as those shown in the curves of Fig. 15 were obtained with the passive element mechanization. The relative magnitude of the summing resistor as compared with the isolation resistors was ad-

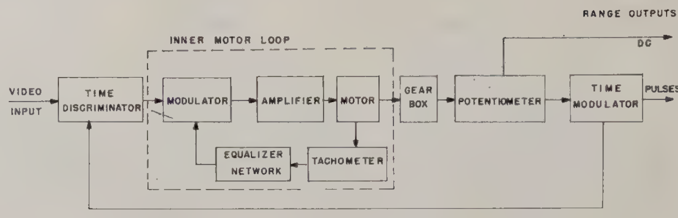

Fig. 19—Moon-tracking automatic radar range system block diagram.

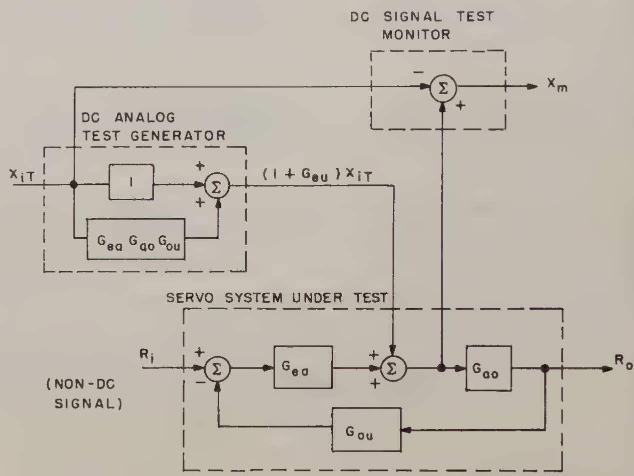

Fig. 20—Test arrangement for servo system with dc signal transmission at some point in the loop.

justed so that the desired error voltage step was obtained from the available dc supply. The negative $(1+G_{eo})K_T$ requirement was resolved by accepting an error negative to that previously obtained for a given dc input polarity. Since the error was observed directly and no comparison with the input was made, no difficulty resulted.

A Practical Application

An early practical application of the analog testing method was during the development of a high-performance automatic range tracking system for radar echoes from the moon.[2] A block diagram of this two-loop system is shown in Fig. 19. In development of the inner motor loop, the analog testing method of Fig. 3 was found directly applicable. Testing of the complete system using a dc-analog signal generator posed difficulties not encountered in all-dc servo system testing. One possible approach involves introduction of converters, *e.g.*, a dc-voltage controlled pulse generator (time modulator) between the signal generator and the video input terminal. The transfer characteristics of these auxiliary converters must be taken into account in performing the test. Another possible approach, shown in Fig. 20, depends upon the servo system under test having signal transmission as varying-dc at some point in the loop (either in the forward or in the feedback path). In this test arrangement, a restriction on the test duration

[2] J. E. Abel and C. F. White, "An automatic radar range moon tracking system," Naval Res. Lab. Report 5170; August, 1958.

arises from holding the normal servo system input at a fixed value. The test is limited to the linear region of the servo system error detector. This restriction precludes use of steady-state information with consequent definition of the extremely low-frequency portions of the servo system transfer function. In the moon tracking servo system example, the test duration found practicable resulted in definition of the transfer function from the unity-gain frequency to a frequency approximately two decades lower.

Adaptive Servo System Possibilities

A careful study of the results obtained using the analog method of servo system performance testing leads to the suggestion that this is a sensing method suited to control of adaptive servo systems. In modern autopilots, for example, many gain changes are introduced as a function of variables that in themselves cannot be accurately measured. By periodic introduction of short duration test signals of the form considered here, electronic circuits designed to sense the monitor test output level and correct the servo gain in the appropriate direction and by the appropriate amount could possibly bypass the current instrumentation problems. The combined effect of all variables influencing the control characteristics would be automatically encompassed. The correction would be to reestablish the transfer function of the servo system to a previously specified characteristic known to be satisfactory. Not only is there promise of loop gain adjustment by this method but also of transfer function corner frequency control.

Conclusions

The basic concept of a new method of servo system performance testing has been presented. The analog testing method detailed has application to linear and nonlinear servo system experimental analysis and system development and to go—no-go production, built-in, fault location, and maintenance testing. Possibilities in the field of adaptive servo systems are indicated. Frequency-domain parameters of the servo system are accurately determined by the use of easily produced transient (time-domain) signal generator input signals. The signal generator used is formed from operational amplifiers of the analog computer type or, alternatively, from entirely passive elements. The results reported indicate that the sensitivity of open-loop testing is achieved in a closed-loop testing procedure.

Appendix A

Approximation to Loop Gain at Zero Frequency

To investigate possible effects of limited dc gain in the test equipment when the transfer-function of the servo system to be tested is not so limited, consider the simple servo system previously studied, using instead

$$G_{eoT} = \frac{\omega_T}{\omega_1} \cdot \frac{\omega_1}{s+\omega_1}$$

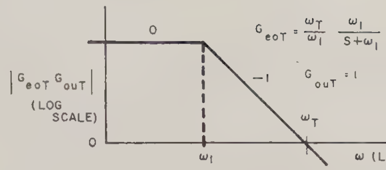

Fig. 21—Asymptotic plot of the approximation to $G_{eo}G_{ou}$ when the signal generator contains operational amplifiers having limited dc gain.

as shown in Fig. 21. The output of the signal generator becomes

$$X_i(s) = \left(1 + \frac{\omega_T}{s+\omega_1}\right)\frac{1}{s}$$

$$X_i(t) = 1 + \frac{\omega_T}{\omega_1}(1 - e^{-\omega_1 t}). \tag{6}$$

At a time $t = 10/\omega_T$, corresponding to 10 system time constants after the initiation of the step-function, the function specified by (6) has a value equal to 99.5 per cent of the correct value if a dc gain (ω_T/ω_1) of 1000 is used in the signal generator analog of the servo system loop gain.

For an input to the servo system of the form given by (6), the error signal becomes

$$G_{eo}(s) = \frac{s+\omega_1+\omega_T}{s(s+\omega_1)} \cdot \frac{1}{1+(\omega_0/s)}$$

$$G_{eo}(t) = \left(\frac{\omega_T}{\omega_0-\omega_1}\right)e^{-\omega_1 t} + \left(1 - \frac{\omega_T}{\omega_0-\omega_1}\right)e^{-\omega_0 t}. \tag{7}$$

For the case of the servo system exactly on specifications, $\omega_0 = \omega_T$ and

$$G_{eo}(t) = \left(\frac{\omega_T}{\omega_T-\omega_1}\right)e^{-\omega_1 t} - \left(\frac{\omega_1}{\omega_T-\omega_1}\right)e^{-\omega_T t}. \tag{8}$$

At a time $t = 10/\omega_T$,

$$G_{eo}(t) = \left(\frac{\omega_T}{\omega_T-\omega_1}\right)e^{-10\omega_1/\omega_T} - \left(\frac{\omega_1}{\omega_T-\omega_1}\right)e^{-10}. \tag{9}$$

If the dc gain of 1000 is used, $\omega_T/\omega_1 = 1000$ and

$$G_{eo}(t) = \frac{1000}{999}e^{-0.01} - \frac{1}{999}e^{-10}. \tag{10}$$

Evaluation of (10) shows that the error is 99.1 per cent of the proper value of unity at 10 system time constants after the start of the step-function.

Appendix B

Passive-Element Signal Generator

The variation of transmission with frequency of the transfer function of Fig. 12 may be approximated by a two-section low-pass resistance-capacitance network of the form shown in Fig. 17(a). If the input impedance of the second section is made high compared with the out-

put impedance of the first section, the transfer function for the network is approximately

$$\frac{e_2}{e_1} = \frac{1}{1 + sCRab} \frac{1 + sCRef}{1 + sCR(d + e)f}. \quad (11)$$

We then define (see Fig. 22)

$\omega_0 = (1/CR)(1/ef)$

$\omega_a =$ low-pass corners at a frequency low compared to ω_0

$$= (1/CR)(1/ab)$$

$$= \frac{1}{CR} \cdot \frac{1}{(d + e)f}. \quad (12)$$

The ratio ω_0/ω_a should be at least $\sqrt{1000/4}$ or equal to about 16 for a dc gain of 1024. That is,

$$\frac{d + e}{e} = 16 \quad \text{or} \quad d = 15e$$

$$\frac{ab}{ef} = 16 \quad \text{or} \quad ab = 16ef. \quad (13)$$

The values given on Fig. 17(b) meet the above requirements. An exact analysis of the transfer function for the network of Fig. 17 shows

$$\frac{e_2}{e_1} = \frac{1 + sCRef}{1 + sCR[ab + (a + d + e)f] + s^2C^2R^2ab(d + e)f}. \quad (14)$$

The double integrator asymptotic segment is defined by

$$\frac{e_2}{e_1} = \frac{1}{s^2C^2R^2ab(d + e)f}. \quad (15)$$

Substituting the frequency of the break between the -2 and the -1 slopes, $\omega_0 = (1/CR)(1/ef)$, we find an attenuation

$$\frac{e_2}{e_1} = \frac{1}{\frac{1}{e^2f^2} ab(d+e)f} = \frac{e^2f}{ab(d + e)}. \quad (16)$$

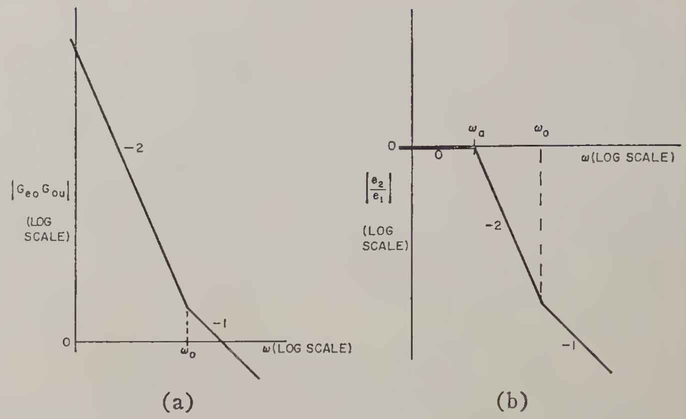

Fig. 22—Passive-element approximations: (a) Range-tracking function, and (b) RC network function.

Substitution of the parameter value from Fig. 17(b) verifies that a dc gain of 1024 is the proper factor for the other portion of the $1 + G_{eo}G_{ou}$ signal generator.

Acknowledgment

L. F. Gilchrist is responsible for the passive-element signal generator idea. In addition to his valuable suggestions and assistance, those of W. Whiting, J. P. Dougherty, C. E. Corum, C. H. Dodge, and J. W. Titus of the Radar Division, Naval Research Laboratory, are gratefully acknowledged. Application to autopilot design was suggested by D. J. Povejsil, Air Arm Division, Westinghouse Corporation.

Considerations in Phase Shifting*

M. G. REKOFF, JR.†

Summary—**The problem of phase shifting is investigated from an ideal viewpoint. The different possibilities of obtaining quadrature voltages at the terminals of an ac servomotor with static impedance elements, connected to the reference winding, are discussed together with the limiting restrictions for each. Formulas for specifying elements of the phase shifting impedance are given in terms of motor constants for several of the more common impedance configurations.**

Introduction

The purpose of this paper is to enumerate several methods of obtaining quadrature voltages at the terminals of a two-phase servomotor from a single phase source using static circuit elements. Several[1–3] papers have been published which give formulas and methods for computing the impedance or impedances required for phase shifting, but these methods generally do not result in voltages in quadrature for all possible source voltages and motor impedance combinations. The discussions presented here are limited solely to the case in which the voltages obtained are in quadrature for a given operating speed of the motor. It is well known that the motor impedance is a function of rotor speed; therefore, use of static impedance elements will give the desired phase shift at only one speed. The reader should note that a zero source impedance is assumed in the following discussion. It is hoped that this discussion will outline a philosophy in the solution of this problem.

Methods of Phase Shifting

The variables in the phase shifting problem are motor speed, the desired magnitude of the voltage applied directly to the motor reference winding, the possibility of using an impedance in series with the motor winding and source, and the possibility of using an impedance in parallel with the motor winding or a series-parallel combination of impedances. The motor speed and the magnitude of voltage applied to the reference winding are usually determined by considerations other than phase shifting. This leaves the designer the possibility of specifying the configuration of the phase shifting impedances (series, parallel, or combination thereof) and the numerical value of each of the elements of this impedance. There are, of course, two possible elements comprising each impedance.

One method of phase shifting, the single impedance method, is to connect an impedance in series with the voltage source and the winding on which the voltage is to be shifted. A second method, called the two impedance method, is to connect one impedance in parallel with the winding under consideration and connect a second impedance in series with the voltage source and the paralleled motor-impedance combination.

Another possible, though not common, variable in the problem is the magnitude of the source voltage itself. This magnitude may be fixed by other considerations or it can be specified by the designer. It may be necessary in some instances to change the available source voltage magnitude for reasons apart from phase shifting. In those cases where a transformer must be used, its turns ratio possibly can be selected to secure optimum conditions for phase shifting. No added expense is involved since the transformer is required in any event. Except for this case it is generally not economically practical to change the source voltage magnitude.

In the following discussion, motor impedance and power factor mean the values of these quantities when the motor is operating at the desired speed.

Single Impedance Method

The schematic diagram for this method of phase shifting is shown in Fig. 1. In this diagram it is assumed that

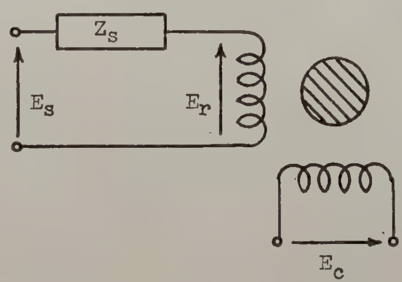

Fig. 1—Single impedance method.

E_s and E_c are in phase (or 180° out) and that E_s and E_r are to be in quadrature. Furthermore, it is also assumed that $|E_s| = N|E_r|$; where N is a real positive constant, E_r is the voltage to be applied to the reference winding, and E_s is the source voltage. If Z_s is defined as $a+jb$, it can be shown that to obtain E_r in quadrature with E_s (hence E_c) and have its magnitude equal to the chosen winding voltage magnitude,

$$a = (Nx_m - r_m) \qquad b = -(Nr_m + x_m) \tag{1}$$

where x_m and r_m are the elements of motor impedance. Obviously $Nx_m \geqq r_m$ must be true in order to have a realizable resistance.

* Manuscript received by the PGAC, January 14, 1958.
† Dept. of Elec. Eng., Univ. of Wisconsin, Madison, Wis.
[1] K. Burian and T. Bottis, "How to operate a two phase motor from a single phase source," *Control Eng.*, vol. 2, p. 48; January, 1955.
[2] S. Davis, "Two capacitor method of phase shifting," *Control Eng.*, vol. 3, p. 71; January, 1956.
[3] M. Rekoff, "Getting maximum torque out of a two phase servomotor," *Control Eng.*, vol. 4, pp. 121–122; June, 1957.

To illustrate some interesting consequences of this, suppose that the chosen motor winding voltage magnitude is equal to the source voltage magnitude ($N=1$); then to have a realizable resistance $x_m \geqq r_m$, which implies that the power factor of the motor must be 0.707 or less to obtain voltages in quadrature. The power factor is

$$Pf = \cos\left(\tan^{-1}\frac{x_m}{r_m}\right).$$

Again, suppose one has available a 110/220 volt system to which a 110-volt motor is to be connected. If the motor power factor is greater than 0.707, quadrature voltages cannot be obtained from a 110-volt source. But with a 220-volt source, $N=2$ which implies that the limiting case for realizability becomes $2x_m = r_m$. This means that the power factor of the motor can be as large as 0.895 and one can still obtain quadrature voltages. A survey of manufacturers' data indicates that a great number of the 60-cps servomotors have power factors which fall into the range 0.707–0.895.

The value of motor power factor which satisfies $Nx_m = r_m$ for the given source and motor winding voltages will be denoted as the limiting power factor. Note that in the single impedance case any power factor larger than the limiting value will require a negative resistance in the shifting impedance. If for a given motor the power factor has exactly the limiting value, then phase shifting can be accomplished using only a capacitor. If the power factor is less than the limiting value then the roles of the elements of the impedance Z_s might be described in the following manner. The resistance serves to bring the power factor of the motor to the limiting value so that the phase shifting can be accomplished by the capacitor.

In some instances the system designer can specify the magnitude of the source voltage. In these cases it appears advisable to accomplish the phase shifting with a single element, namely the capacitor. By proper algebraic manipulation of the expressions in (1), N can be chosen on the basis of the motor power factor so that a single capacitor can be used to shift the phase. The expression for this is

$$N = \frac{Pf}{\sqrt{1 - Pf^2}}. \tag{2}$$

The reasoning in this instance is that the source voltage is adjusted so that the motor power factor will be the limiting power factor and the phase shift is accomplished by a capacitor. There is a practical limit to this process since a hypothetical motor having $Pf=1$ would require an infinite source voltage.

Two Impedance Method

The schematic for this method of phase shifting is shown in Fig. 2. The same relations between E_r, E_c, and E_s are assumed as were used for the single impedance method. The impedance Z_s is defined as $a+jb$ and Z_p as $c+jd$.

Generally it is advisable to use purely reactive elements as phase shifting elements to reduce the heating and power loss, and to simplify the expressions for the unknown elements. Setting the resistive components to zero will define $Z_s = +jb$ and $Z_p = +jd$. It can be shown that to obtain quadrature voltage at the terminals of the motor and for E_r to be of the proper magnitude

$$b = -\frac{N(r_m^2 + x_m)^2}{r_m} \qquad d = -\frac{N(r_m^2 + x_m^2)}{Nx_m - r_m}. \tag{3}$$

Note that if $Nx_m = r_m$ the motor has the limiting power factor and the impedance required is reduced to only a capacitor in series with the source and the motor. Note that the limiting power factor in this case does not impose a theoretical limit on design as before, since if $r_m > Nx_m$, this merely means that the reactance d should be inductive instead of capacitive.

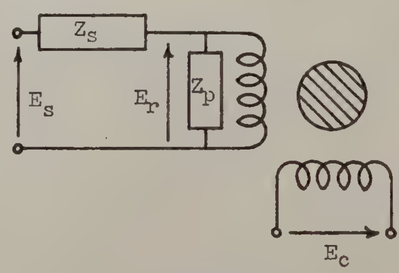

Fig. 2—Two impedance method.

Assuming ideal reactive elements means that quadrature voltages can be obtained on a motor of any power factor by the use of just two reactive elements. The reasoning in this instance is that if the power factor of the motor is smaller than the limiting power factor, a capacitor is connected in parallel with the motor winding to raise the power factor of the paralleled combination to the limiting power factor so that phase shifting can be accomplished using only a capacitor. If the power factor of the motor is greater than the limiting power factor, an inductance (pure) is connected in parallel with the motor winding to lower the power factor of the parallel combination to the limiting value so that the phase shifting can be again accomplished using only a capacitor.

It is, of course, impossible to secure an ideal inductance. Therefore, in those cases where one is called for above, the actual procedure is changed somewhat. In this instance $Z_p = c+jd$, and if $Z_s = -jb$ ($a=0$), one can express the relation between c and d in terms of the Q of the inductance coil. If Q is defined as the ratio of the reactance to the resistance, the reactance being determined at the carrier frequency

$$Q = \frac{d}{c} \tag{4}$$

then

$$c = \frac{(NQ - 1)(x_m^2 + r_m)^2}{(1 + Q^2)(r_m - Nx_m)} \tag{5}$$

and

$$b = \frac{(r_m^2 + x_m^2)(Nc + d) + (c^2 + d^2)(Nr_m + x_m)}{(r_m + c)^2 + (x_m + d)^2}. \tag{6}$$

Note that for c to be realizable, $r_m > Nx_m$; or the power factor of the motor *must* be greater than the limiting power factor. This, however, is actually no restriction, for if the motor power factor is less than the limiting value, the methods discussed previously are applicable.

If the expression for d in (3) implies that a capacitor be used for Z_s, one has an alternative of using a resistance rather than the capacitor.

If the power factor of the motor is smaller than the limiting power factor, one may use a resistance in parallel with the motor winding and a capacitor in series with the source and the paralleled resistance and motor. The reasoning in this case is that the resistance will improve the power factor of paralleled combination to the limiting value so the phase shift can be obtained using only a capacitor. Designating $Z_s = +jb$ $(a=0)$ and $Z_p = c(d=0)$, one obtains the following specified c and b

$$c = \frac{r_m^2 + x_m^2}{Nx_m - r_m} \tag{7}$$

$$b = -\frac{(r_m^2 + x_m^2)[N^2 - 1]x_m}{(Nx_m - r_m)^2 + 2r_m(Nx_m - r_m) + (r_m^2 + x_m^2)}. \tag{8}$$

Although this possibility exists one would normally prefer to use the capacitor in this case to avoid the power loss associated with the resistor.

Any other possible combination of impedances can be investigated by application of the general expressions developed in the Appendix.

Conclusion

From the preceding discussion it appears that there are several methods for phase shifting in any given case. The designer must determine which is best suited to his particular problem considering not only the problem of phase shifting but also other effects of his choice on system performance. Also, from an economic standpoint, it may not be desirable to insure quadrature voltages, since for most purposes the phase angle between the voltages may be somewhat less than 90 degrees and a system will still be satisfactory. The problem becomes more involved if the servomotor is to be operated over a wide range of speeds.

The author is currently occupied with investigating and evaluating the effect of various methods of phase shifting upon the servomotor performance. Items of importance appear to be motor terminal voltage, volt-ampere requirements from the source, and effect of phase shifting impedance on internal damping of the motor.

Appendix

Obtaining Impedance Expressions

The expressions in this appendix were originally worked out with two phase symmetrical components. However, a derivation of this type is rather involved, hence a circuit approach is used here. $Z_m = r_m + jx_m$ is the per phase motor impedance when the motor is operating under balanced conditions with rated voltage applied and is running at the desired speed. Since the condition of rated voltages in quadrature at the motor terminals is specified (balanced conditions), the following derivation is valid. For any other condition symmetrical components must be resorted to.

Single Impedance Method

Referring to Fig. 1,

$$|E_s| = N|E_r|$$

$$E_r = Z_m \frac{E_s}{Z_m + Z_s}$$

but

$$E_r = E\underline{/90°}$$

$$E_s = NE\underline{/0°}$$

$$E\underline{/90°} = \frac{Z_m}{Z_m + Z_s} NE\underline{/0°}$$

$$j = \frac{Z_m}{Z_m + Z_s} N$$

if

$$Z_s = a + jb$$

and

$$Z_m = r_m + jx_m$$

$$ja - b + jr_m - x_m = Nr_m + jNx_m$$

$$\left.\begin{aligned} a &= Nx_m - r_m \\ b &= -(Nr_m + x_m) \end{aligned}\right\}. \tag{9}$$

To specify N so that phase shift can be accomplished with only a capacitor let

$$a = 0$$

$$0 = Nx_m - r_m$$

or

$$N = \frac{r_m}{x_m}$$

from the impedance triangle (Fig. 3).

$$\theta = \tan^{-1} \frac{x_m}{r_m}.$$

Motor Power factor $= Pf = \cos\theta = \cos\tan^{-1} x_m/r_m$
or

$$\frac{x_m}{r_m} = \tan\cos^{-1} Pf$$

but

$$\cos^{-1} m = \tan^{-1}\frac{\sqrt{1-m^2}}{m};$$

therefore

$$\frac{x_m}{r_m} = \tan\tan^{-1}\frac{\sqrt{1-Pf^2}}{Pf} = \frac{\sqrt{1-Pf^2}}{Pf} = \frac{1}{N}$$

or

$$N = \frac{Pf}{\sqrt{1-Pf^2}}. \tag{10}$$

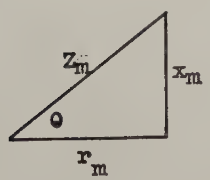

Fig. 3—Impedance triangle.

Two Impedance Method

Referring to Fig. 2,

$$E_r = \frac{Z_mZ_p}{Z_m + Z_p} \frac{E_s}{Z_s + \frac{Z_mZ_p}{Z_m+Z_p}}.$$

Again

$$E_r = E\underline{/90°}$$

and

$$E_s = NE\underline{/0°}$$

$$E\underline{/90°} = \frac{NE\underline{/0°}Z_mZ_p}{Z_sZ_m + Z_sZ_p + Z_pZ_m}$$

if

$$Z_s = a + jb,$$
$$Z_p = c + jd,$$

and

$$Z_m = r_m + jx_m,$$

$$j = \frac{N(r_m+jx_m)(c+jd)}{(a+jb)(c+jd)+(a+jb)(r_m+jx_m)+(r_m+jx_m)(c+jd)}.$$

Expanding and separating into real and imaginary parts,

$$a(r_m + c) - b(x_m + d) = Ncx_m + Nr_md + x_md - r_mc$$
$$a(x_m + d) + b(r_m + c) = -Nr_mc + Ndx_m - x_mc - r_md.$$

Solving for a and b,

$$\left.\begin{aligned} a &= \frac{(Nd-c)(r_m^2+x_m^2) + (Nx_m - r_m)(c^2+d^2)}{(r_m+c)^2 + (x_m+d)^2} \\ b &= -\frac{(r_m^2+x_m^2)(d+Nc) + (c^2+d^2)(x_m+Nr_m)}{(r_m+c)^2+(x_m+d)^2} \end{aligned}\right\}. \tag{11}$$

Using the expressions in (11) one can develop the impedance relations by assuming appropriate values for chosen parameters. For example, to obtain the equations in (3) one sets $a=0$ and $c=0$ and determines the relations for b and d from (11).

It is important to note that the impedance Z_s necessarily contains the source impedance in the practical case since the development of the expressions assumed a zero source impedance.

Analysis of Gyro Orientation*

ARTHUR MAYER†

Summary—**The frequency response of a gyro-stabilized platform is limited by the gyros. For the most effective suppression of instability with a minimum sacrifice in bandwidth, it is necessary to orient single-degree-of-freedom gyros in a certain way. Methods of orientation are discussed in this paper.**

Definitions

THIS paper considers stability problems involved in the orientation of gyros on a stable platform in an aircraft or other vehicle. The ultimate analysis is based on a three-degree-of-freedom system, involving three single-degree-of-freedom, floated, integrating gyros (shown in Fig. 1) mounted on the platform being stabilized. The results show that it is generally best to have all three gyros placed with their output axes horizontal. For the heading gyro there is no other choice, since the input axis is vertical, but the other two gyros have been so oriented to preserve stability in the gimbal-drive servos under all conditions. Otherwise instability would sometimes develop when the aircraft banks steeply.

In order to discuss this instability, it is convenient to consider a two-gimbal system placed with the axis of rotation of the outer gimbal parallel to the longitudinal axis of the aircraft, and with two gyros mounted on the inner gimbal. It is assumed that the input axes of the two gyros are perpendicular to each other, and that it is desired to keep both input axes horizontal.

Let θ_g be the angle of rotation of the inner gimbal, and let ϕ_g be the angle of rotation of the outer gimbal. In order to provide a reference from which to measure θ_g and ϕ_g, consider aircraft pitch and roll to be zero and both gyro input axes to be horizontal, and assume that θ_g and ϕ_g are zero in this condition.

The axis of the inner gimbal will be called the "θ axis." Assume that one gyro, to be called the "θ gyro," has its input axis parallel to the θ axis. The other gyro will be called the "ϕ gyro." Fig. 2(a) illustrates the given orientation.

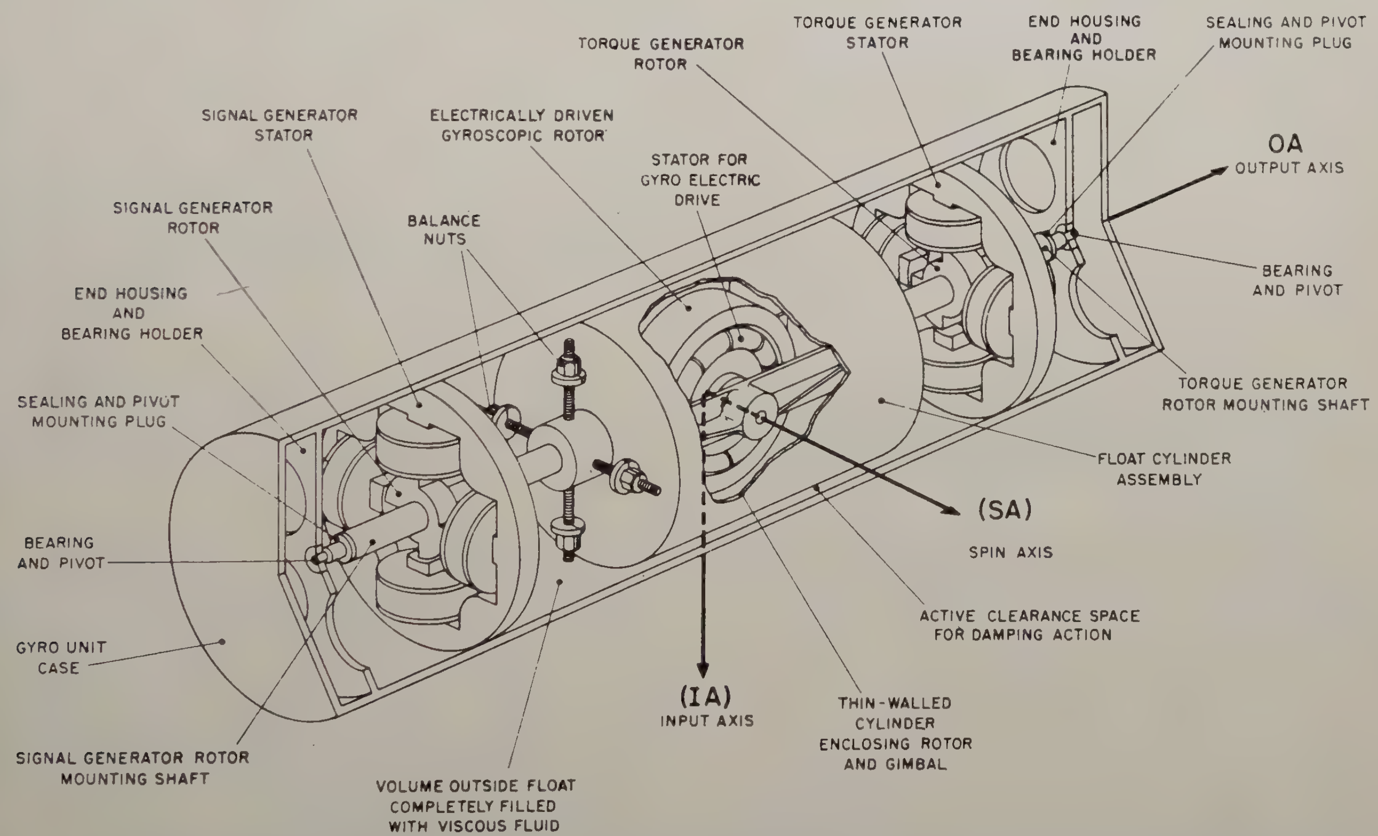

Fig. 1—Cutaway drawing of a floated, integrating gyro. The signal generator senses the angle of rotation of the "float cylinder assembly" about the output axis. The outer case of the gyro is a cylinder whose axis is coincident with the output axis.

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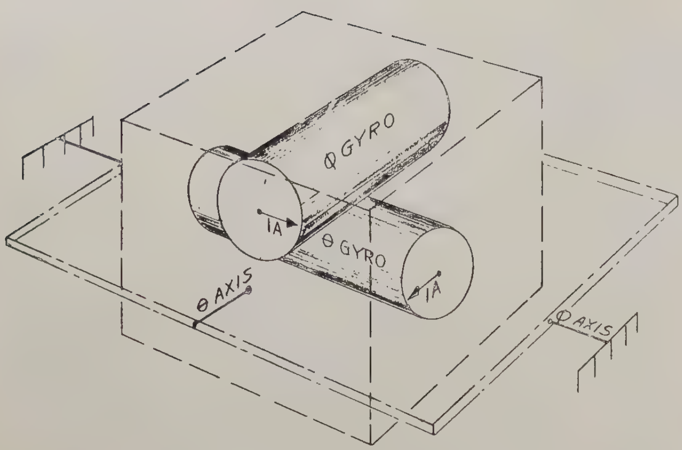
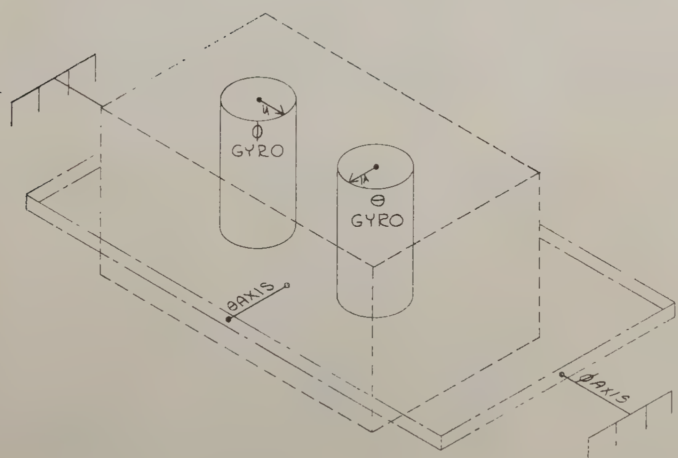


(a)



(b)

Fig. 2—Two-gimbal inertial platform. The directions of the input axes, designated IA, are the same in (a) and (b), but the directions of the output axes are different. These are two of the many possible orientations that the gyros may have with this choice of input axes. The most commonly used orientation is shown in (b), although this analysis indicates that the orientation shown in (a) is usually preferable.

If θ_a and ϕ_a are aircraft pitch and roll, respectively, then both gyro input axes will be horizontal if $\theta_g = -\theta_a$ and $\phi_g = -\phi_a$.

Accordingly, let

$$\epsilon_\theta = \theta_a + \theta_g$$
$$\epsilon_\phi = \phi_a + \phi_g. \tag{1}$$

The gimbals are to be servo driven to make ϵ_θ and ϵ_ϕ zero.

The inner gimbal, to which the θ and ϕ gyros are rigidly attached, has an angular velocity due to motion of the aircraft and of the two gimbals. This angular velocity can be represented by a vector, which may be resolved into components along the two horizontal input axes and the vertical. Let ω_θ, ω_ϕ, and ω_ψ be the components of angular velocity in the respective directions of the θ-gyro input axis, the ϕ-gyro input axis, and the vertical. Then

$$\omega_\theta = \dot{\epsilon}_\theta$$
$$\omega_\phi = \dot{\epsilon}_\phi \cos \theta_g$$
$$\omega_\psi = \dot{\epsilon}_\psi \sin \theta_g + \dot{\psi}_a \tag{2}$$

where ψ_a is the heading angle of the aircraft. These equations are valid only when ϵ_θ and ϵ_ϕ are small; that is, when the two input axes are nearly horizontal.

Ideal Gyros

The output signal of an ideal gyro—that is, a gyro in which the moment of inertia about the output axis is zero—is proportional to the integral of the angular velocity component along the input axis. Moreover, an ideal gyro is insensitive to the angular velocity or acceleration components along the output axis or along the spin axis. If δ_θ and δ_ϕ are the output angles of the θ and ϕ gyros, respectively, then

$$c\dot{\delta}_\theta = H\omega_\theta$$
$$c\dot{\delta}_\phi = H\omega_\phi \tag{3}$$

where H is the angular momentum of the gyro wheel and c is the damping constant of the gyro. It follows from (2) and (3) that

$$\dot{\delta}_\theta = \frac{H}{c}\dot{\epsilon}_\theta$$
$$\dot{\delta}_\phi = \frac{H}{c}\dot{\epsilon}_\phi \cos \theta_g. \tag{4}$$

The output signal of the θ gyro is amplified and fed back to drive the inner gimbal. If the rate at which the servo motor drives is proportional to the gyro output angle, then

$$\dot{\theta}_g = -G\delta_\theta \tag{5}$$

where G is a gain factor. The transform of (5) is

$$s\theta_g = -G\delta_\theta = -G\frac{H}{c}\epsilon_\theta \tag{6}$$

and the servo loop gain is

$$-\frac{\theta_g}{\epsilon_\theta} = \frac{G}{s}\frac{H}{c}. \tag{7}$$

The servo that drives the inner gimbal will be called the "θ servo."

The output signal of the ϕ gyro is amplified and fed back to drive the outer gimbal. If the rate at which the servo motor drives is proportional to the gyro output angle, and is also proportional to sec θ_g, then

$$\dot{\phi}_g = -G\delta_\phi \sec \theta_g. \tag{8}$$

For any constant θ_g, the transform of (8) is

$$s\phi_g = -G\delta_\phi \sec \theta_g = -G\frac{H}{c}\epsilon_\phi \tag{9}$$

and the servo loop gain is

$$-\frac{\phi_g}{\epsilon_\phi} = \frac{G}{s}\frac{H}{c}. \tag{10}$$

The servo that drives the outer gimbal will be called the "ϕ servo."

Because of the inertia of the servo motors, and perhaps because of the characteristics of the servo amplifiers, the rate at which the gimbals rotate will not be strictly proportional to the gyro output angles. Eqs. (6), (7), (9), and (10) will still remain valid, but the motor and amplifier characteristics have the effect of making G a function of s. If the function $G(s)$ is known, the stability and the response of the servos may be inferred from their loop gains (7) and (10).

Practical Gyros (Two-Gimbal System)

If either gyro case is removed from its mounting, rotated about the input axis through some angle, and then replaced, the preceding equations will remain unaltered despite the change in direction of the output axis of the gyro, and hence the behavior of the system will be unaffected. However the equations are idealized in that they assume the moment of inertia of each gyro about its output axis to be zero. In an actual gyro this inertia causes the gyro to lag in its response to rotations about the input axis, and also causes the gyro to respond to angular accelerations about the output axis. The former effect can be incorporated in G along with the motor and amplifier characteristics. The latter effect, however, is not so easily eliminated. Because of it, the behavior of the system does depend upon the directions of the gyro output axes.

Derivation of Loop Equations

Let δ be the output angle of a gyro, and let ω_i and ω_o be the components of angular velocity along the input and output axes, respectively. Then

$$J\ddot{\delta} + c\dot{\delta} = H\omega_i - J\dot{\omega}_o \tag{11}$$

where J is the moment of inertia of the gyro about its output axis, H is the angular momentum of the gyro wheel, and c is the damping constant of the gyro. Eq. (11) implies a certain convention as regards positive and negative rotations, but no restriction is imposed by the convention; for example, the effect of changing the sign of δ may be achieved by reversing the phase of the excitation on the gyro signal generator, and the sign of H may be changed by reversing the direction of rotation of the gyro wheel.

Let α be the angle between the θ-gyro output axis and the ϕ-gyro input axis, and let β be the angle between the ϕ-gyro output axis and the θ-gyro input axis. [In Fig. 2(a), α and β are either 0° or 180°.][1] The components of angular velocity along the two output axes are

$$\begin{aligned}\omega_{0,\theta} &= \omega_\phi \cos\alpha + \omega_\psi \sin\alpha \\ \omega_{0,\phi} &= \omega_\theta \cos\beta + \omega_\psi \sin\beta.\end{aligned} \tag{12}$$

[1] In Fig. 2(b), α and β are either 90° or 270°.

From (11) it can be seen that (3) is incomplete and should be replaced by

$$\begin{aligned}J\ddot{\delta}_\theta + c\dot{\delta}_\theta &= H\omega_\theta - J(\dot{\omega}_\phi \cos\alpha + \dot{\omega}_\psi \sin\alpha) \\ J\ddot{\delta}_\phi + c\dot{\delta}_\phi &= H\omega_\phi - J(\dot{\omega}_\theta \cos\beta + \dot{\omega}_\psi \sin\beta).\end{aligned} \tag{13}$$

Eq. (13) is the practical gyro equivalent of (3) for the ideal gyro. From (2) and (13), the transforms of δ_θ and δ_ϕ are found to be

$$\delta_\theta = \frac{H\epsilon_\theta - Js\epsilon_\phi \cos(\theta_g - \alpha) - Js\psi_a \sin\alpha}{c + Js}$$

$$\delta_\phi = \frac{(H\cos\theta_g - Js\sin\theta_g \sin\beta)\epsilon_\phi - Js\epsilon_\theta \cos\beta - Js\psi_a \sin\beta}{c + Js}. \tag{14}$$

It is assumed in (14) that the factors $\cos\theta_g$ and $\sin\theta_g$ that appear in (2) are quasi-static. This assumption simplifies the ensuing analysis considerably without materially affecting its validity.

The loop gain of the θ servo can be found from (14) by setting $\epsilon_\phi = 0$ and $s\psi_a = 0$, forming the quotient $\delta_\theta/\epsilon_\theta$, and multiplying that quotient by the gain of the servo amplifier and motor. Accordingly, the θ-servo loop gain is

$$G_\theta = \frac{G'}{s}\frac{H}{c} \tag{15}$$

where

$$G' = \frac{G}{1 + \dfrac{J}{c}}.$$

The loop gain (15) has the same form as the loop gain (7) which assumed an ideal gyro.

The loop gain of the ϕ servo can be found from (14) by setting $\epsilon_\theta = 0$ and $s\psi_a = 0$, forming the quotient $\delta_\phi/\epsilon_\phi$, and multiplying that quotient by the gain of the servo amplifier and motor. As in the derivation of (10), it is important to make the gain proportional to $\sec\theta_g$. Accordingly, the ϕ-servo loop gain is

$$G_\phi = \frac{G'}{s}\frac{H}{c}\left(1 - \frac{Js}{H}\tan\theta_g \sin\beta\right). \tag{16}$$

The loop gain (16) will have the same form as the loop gain (10) only if $\sin\beta = 0$; that is, only if the output axis of the ϕ gyro is parallel to the input axis of the θ gyro. Otherwise, the loop gain of the ϕ servo will be a function of θ_g. The interrelation between the gyro loops is indicated in Fig. 3.

Stability of ϕ Servo

The stability of the ϕ servo depends upon the phase margin of G_ϕ. Fig. 4 is a Bode diagram for a typical $G'H/sc$ and a typical Js/H. The phase margin is ample when $\theta_g = 0°$, for the graph of $G'H/sc$ has a long section with a slope of 6 db per octave where it crosses the 0-db line.

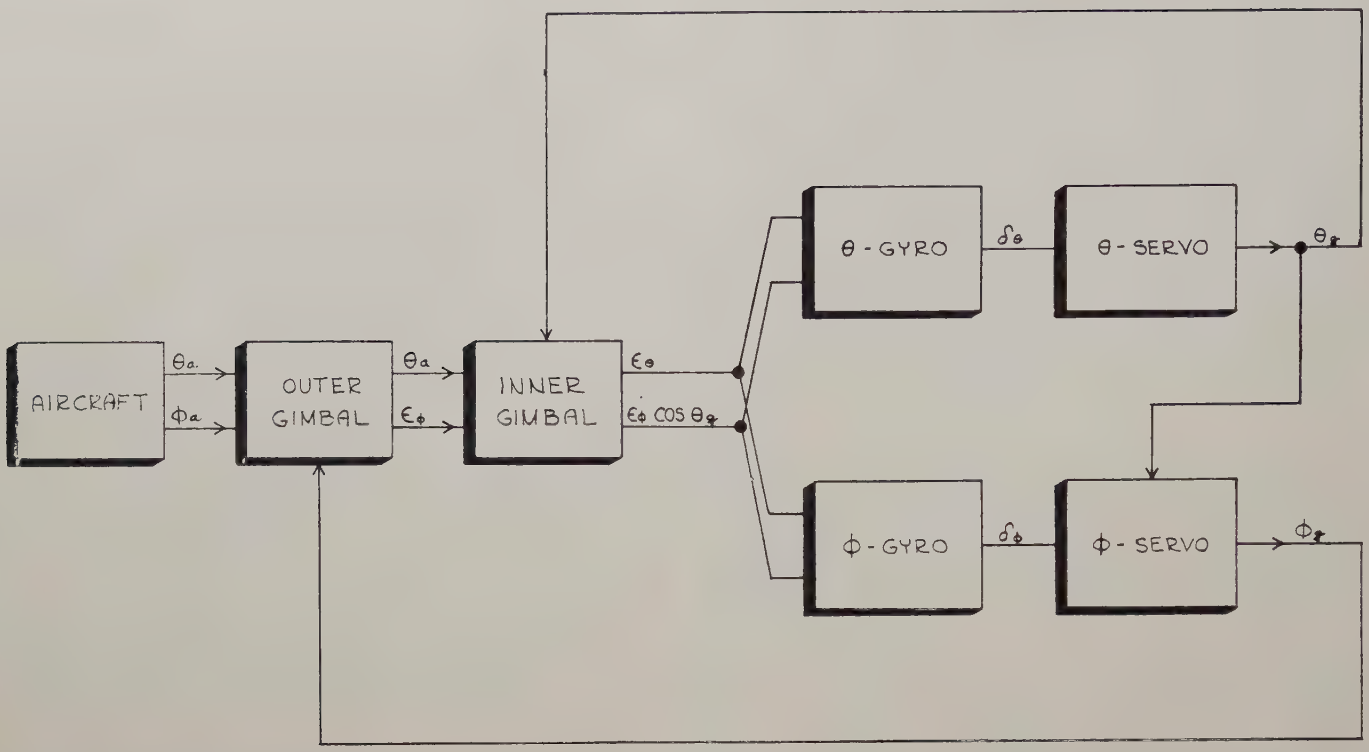


Fig. 3—Block diagram—two-gimbal system. Angular motion of the inner gimbal about the input axis of the θ gyro is shown as ϵ_θ. It affects the θ gyro primarily, but also affects the ϕ gyro through its output axis if the gyros are oriented as in Fig. 2(a). Similarly, angular motion of the inner gimbal about the input axis of the ϕ gyro is shown as $\epsilon_\phi \cos \theta_0$, and may have a secondary effect upon the θ gyro.

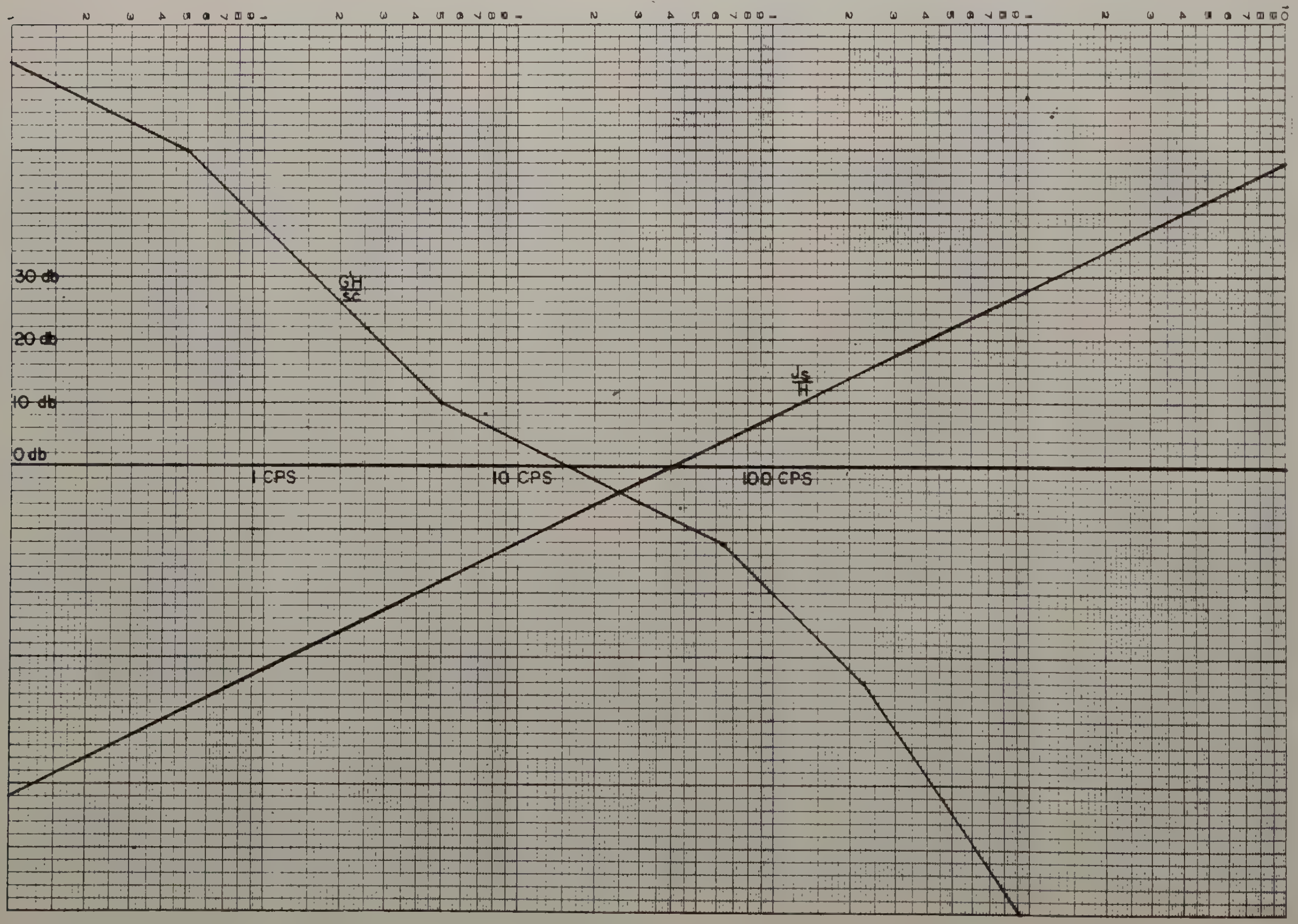


Fig. 4—Gain curves for typical $G'H/sc$ and Js/H. At very low frequencies the servo loop gain has a slope of 6 db/octave, but the slope is made to rise to 12 db/octave at the lowest practical frequency. The gain curve is shaped to reduce the slope to 6 db/octave as it crosses the 0-db line, and then to resume at 12 db/octave. At high frequencies the gain curve may be expected to fall off at 18 db/octave or faster.

The term $-(Js/H)\tan\theta_g\sin\beta$ modifies G_ϕ at high frequencies. Assume, for example, that $\beta=90°$. When θ_g is negative, a phase lead is added to G_ϕ, and as θ_g takes on larger negative values, the lead comes in at lower frequencies. Although the phase margin increases at first, the lead will eventually come in at a low enough frequency to bring the graph of G_ϕ above the 0-db line at frequencies above which the slope of G_ϕ, even with the added lead, is 12 db per octave or greater. Then the phase margin will decrease, and the servo will become unstable as θ_g approaches $-90°$.

When θ_g is positive, a phase lag is added to G_ϕ, and the phase margin decreases as θ_g increases. This does not imply that the graph of G_ϕ will fall more steeply; on the contrary the graph will be raised, exactly as it is raised by the lead which is introduced when θ_g is negative

For the typical G_ϕ and Js/H shown in Fig. 4—if it is assumed that $\sin\beta=1$—when $\tan\theta_g=-1$ $(\theta_g=-45°)$, a lead will be introduced at 40 cps, and the phase margin where the gain curve crosses the 0-db line at 16 cps will increase above its value when $\theta_g=0°$. But when $\tan\theta_g=-10$ $(\theta_g=-85°)$, the lead will come in at 4 cps; the point at 256 cps, which is 6 octaves away, will be raised 36 db, and the graph of G_ϕ will cross the 0-db line at about the frequency where it breaks from a slope of 6 db per octave to a slope of 12 db per octave. The phase margin will be roughly 45°, and will decrease rapidly when θ_g goes beyond $-85°$. (In practice, instability might not develop because the AGC which is responsible for making the gain proportional to $\sec\theta_g$ will cease to function as θ_g approaches $-90°$.)

For positive θ_g, if $\tan\theta_g=1(\theta_g=45°)$, the phase margin at 16 cps will be reduced by $\tan^{-1} 16/40=22°$. When $\tan\theta_g=2.5$ $(\theta_g=68°)$ the phase margin will be reduced by 45°, and it will decrease further when θ_g goes beyond 68°.

The total stable range of the ϕ servo, then, is roughly from $\theta_g=-85°$ to $\theta_g=+65°$. Alternatively, the stable range could be from $-65°$ to $+85°$; but it could not by simple means be extended to 85° in both the positive and negative directions, so long as $\beta=90°$ (or 270°).

Nutation

If the output axis of the ϕ gyro is parallel to the input axis of the θ gyro, as in Fig. 2(a), the stability and the general performance of the ϕ servo will be independent of θ_g, so long as the AGC is operative. However, because of the presence of $\epsilon_\theta\cos\beta$ in (14) for δ_ϕ, the ϕ gyro, and hence the ϕ servo, will be sensitive to inputs that should properly affect the θ servo only.

The over-all transfer function of the ϕ servo for an input ϕ_a, ϵ_θ being identically zero, is

$$-\frac{\phi_g}{\phi_a}=\frac{G_\phi}{1+G_\phi}. \tag{17}$$

If $\dot{\phi}_a$, instead of ϵ_θ, is identically zero, then

$$\frac{\phi_g}{\epsilon_\theta}=\frac{Js\cos\beta}{H\cos\theta_g-Js\sin\theta_g\sin\beta}\cdot\frac{G_\phi}{1+G_\phi}. \tag{18}$$

Similar equations apply to the θ servo. The over-all transfer function of the θ servo for an input θ_a, ϵ_ϕ being identically zero, is

$$-\frac{\theta_g}{\theta_a}=\frac{G_\theta}{1+G_\theta}. \tag{19}$$

If $\dot{\theta}_a$, instead of ϵ_ϕ, is identically zero, then

$$\frac{\theta_g}{\epsilon_\phi}=\frac{Js\cos(\theta_g-\alpha)}{H}\cdot\frac{G_\theta}{1+G_\theta}. \tag{20}$$

Eqs. (17) and (19) show that the responses of the ϕ servo and of the θ servo to inputs ϕ_a and θ_a, respectively, are somewhat imperfect. The resultant errors in ϕ_g and θ_g are augmented, as indicated by (1), (18), and (20), by the response of the ϕ servo to an input θ_a, and of the θ servo to an input ϕ_a. These errors are greatest during rapid changes in θ_a and ϕ_a, and are largely eliminated by the servos when θ_a and ϕ_a settle down to slowly changing values, although some error remains because of the approximations involved in (2).

The consideration of computational errors, although important, is not the subject of the present discussion. Eqs. (1), (18), and (20) show that ϕ_g responds to changes in θ_g, and θ_g responds to changes in ϕ_g. In other words, the ϕ servo and the θ servo together form a servo loop. This servo loop could be an unstable one, and if it is, the servos will oscillate together at some characteristic frequency, even though each servo may be stable by itself. The word "nutation" has been applied to this kind of oscillation.

The loop gain $G_{\theta\phi}$ through the two servos is the negative product of (18) and (20):

$$G_{\theta\phi}=-\frac{(Js)^2\cos(\theta_g-\alpha)\cos\beta}{H(H\cos\theta_g-Js\sin\theta_g\sin\beta)}\cdot\frac{G_\theta}{1+G_\theta}\,\frac{G_\phi}{1+G_\phi}. \tag{21}$$

If $\cos\beta=0$, then $G_{\theta\phi}=0$; but then, as it has been shown, the range of θ_g must be restricted somewhat in order to keep the ϕ servo stable. If $\sin\beta=0$ and $\sin\alpha\neq 0$, then $G_{\theta\phi}$ is a function of θ_g, and nutation will occur as θ_g approaches $\pm 90°$. Accordingly, let $\alpha=0°$ and $\beta=180°$, or let $\alpha=180°$ and $\beta=0°$, so that

$$G_{\theta\phi}=\left(\frac{Js}{H}\right)^2\cdot\frac{G_\theta}{1+G_\theta}\cdot\frac{G_\phi}{1+G_\phi}. \tag{22}$$

From (15) and (16) it is seen that G_θ and G_ϕ both have the simple form $G'H/sc$. However, it is unlikely that the inertia of both servos will be the same; hence the factor G will be different for each of the two servos, and G_θ will not, in general, be equal to G_ϕ.

Fig. 5 is a Bode diagram showing the development of $G_{\theta\phi}$ from the previously assumed G_ϕ and Js/H, and from a G_θ which is assumed to be somewhat greater than G_ϕ. Since the graph of $G_{\theta\phi}$ is always below the 0-db line, the servos will not nutate.

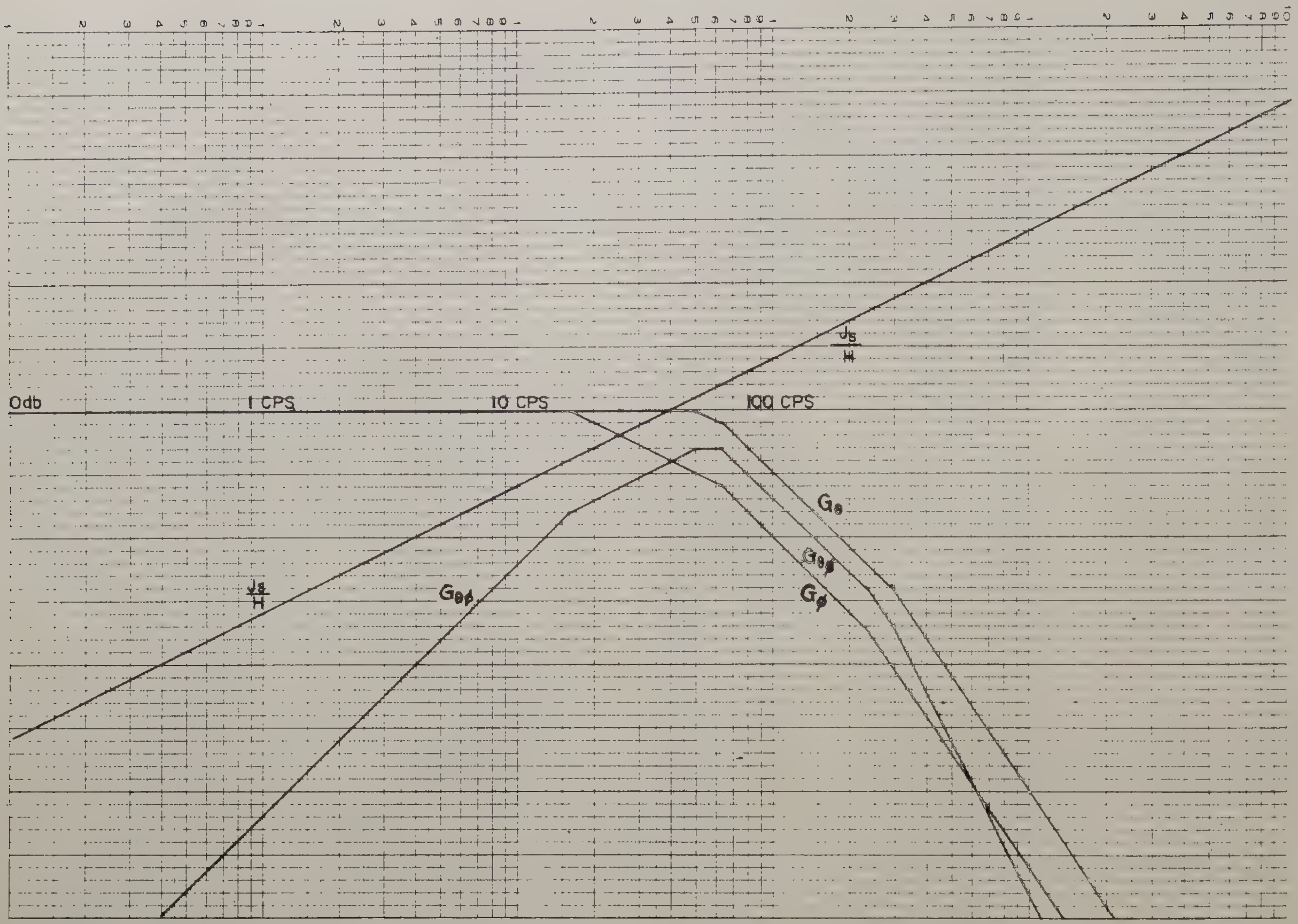


Fig. 5—Gain curves showing development of $G_{\theta\phi}$. The graph of $G_\phi/(1+G_\phi)$ is equal to 1 (or 0 db) at the lower frequencies, where $G_\phi > 0$ db. It is equal to G_ϕ at the higher frequencies, where $G_\phi < 0$ db. The graph of $G_\theta/(1+G_\theta)$ has been constructed in the same way. The graph of $G_{\theta\phi}$ is found from (22) by adding twice the graph of Js/H to the sum of the graphs of $G_\phi/(1+G_\phi)$ and $G_\theta/(1+G_\theta)$.

If both G_θ and G_ϕ cross the 0-db line at a higher frequency than Js/H crosses it, then the graph of $G_{\theta\phi}$ will rise above the 0-db line, and will cross the line, both rising and falling, with a slope of 12 db per octave. In order to avoid nutation, the geometric mean of the crossing frequencies of G_θ and G_ϕ should be less than the crossing frequency of Js/H.

According to the graph of Fig. 5, nutation would not occur even if α and β were both 0° or both 180°, for, since the magnitude of $G_{\theta\phi}$ is always less than 1, its sign is unimportant. However, if an attempt is made to improve servo performance by raising the loop gains G_θ and G_ϕ, nutation would develop as soon as $G_{\theta\phi}$ rose to the 0-db line. It is preferable to have $\alpha = 0°$ and $\beta = 180°$, or $\alpha = 180°$ and $\beta = 0°$; then $G_{\theta\phi}$ could be allowed to rise a little above the 0-db line.

From the above considerations, it is concluded that the gyros should be oriented as shown in Fig. 2(a). This orientation, in which $\sin\beta = 0$, provides for stable operation of the ϕ servo over the maximum possible range of θ_g, and eliminates spurious motions of the gimbals caused by heading rates [see (14)]. Also, since $\sin\alpha = 0$, nutation can be controlled.

Some systems have been built in which α and β are either 90° or 270°. This orientation avoids the possibility of nutation, but it is not justified if the system must work as well for θ_g close to 90° as it does for θ_g close to $-90°$, or if it must undergo changes in heading during operation.

Three-Gimbal System

Ideal Gyros

The entire discussion so far has dealt with a two-gimbal system, in which the inner gimbal constitutes a horizontal platform. When a third gimbal is mounted on the horizontal platform, and is stabilized in heading, the third gimbal constitutes an inertial platform. This is indicated in Fig. 6. It is heading-stabilized by a servo which includes a gyro and a gimbal drive. The detailed operation of this servo will not be considered here.

Since the third gimbal is mounted inside the inner gimbal, it is advisable to rename the outer and inner gimbals, and to call them the "first" and "second" gimbals, respectively. Now, let the θ and ϕ gyros be removed from the second gimbal and mounted on the third gim-

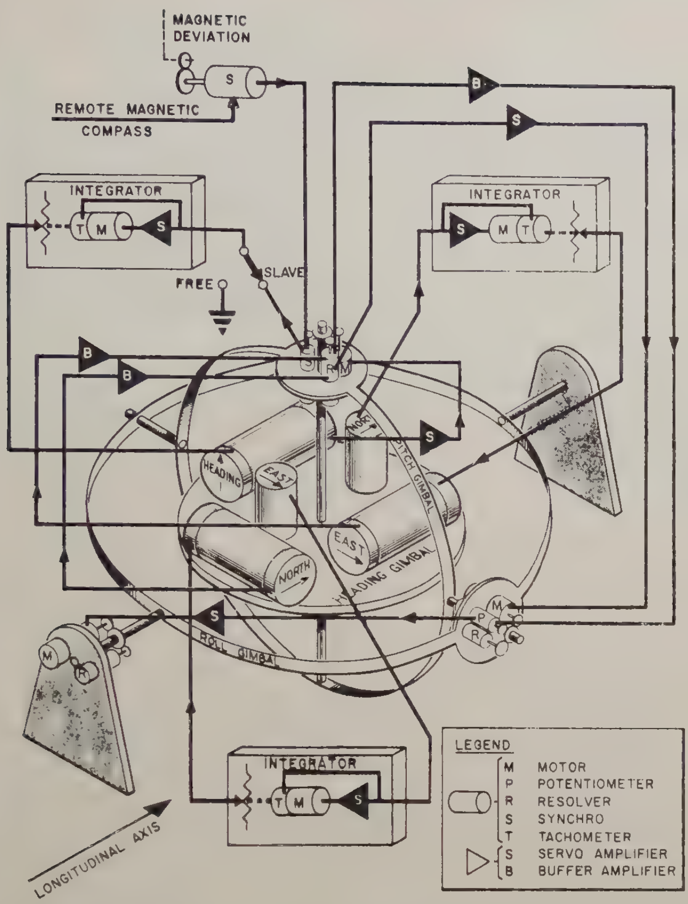


Fig. 6—Complete three-gimbal inertial platform. This figure is a combined block diagram and mechanical schematic. In addition to the three gyros and the three gimbals, with their servo amplifiers and motors, two accelerometers and a magnetic compass are also shown. These operate through the gyro torque generators (see Fig. 1) to maintain the platform horizontal and correctly oriented relative to north for long periods of time.

bal. Since the angles between the θ axis and the two gyro input axes now depend upon heading, neither the θ-servo error signal nor the ϕ-servo error signal can be obtained from a single gyro. Therefore, it is advisable to rename the two gyros whose input axes are horizontal; they shall be called the x gyro and the y gyro.

The effect of gyro orientation on the θ and ϕ servos will now be reviewed, considering the gyros to be on the third gimbal instead of on the second gimbal. One of the advantages of having the gyro output axes horizontal disappears: except for transients, errors in θ_g and ϕ_g are not produced by heading rates, no matter what orientation the gyros may have, because anything on the third gimbal keeps a constant heading. However, the stability of the ϕ servo is still dependent upon θ_g unless the output axes are horizontal.

In order to control the θ and ϕ servos, the output signals from the x and y gyros must be resolved through heading. Let ψ_g be the angle of rotation of the third gimbal. For a heading reference, consider ψ_g to be zero when the x and y gyros have the same orientation with respect to the θ axis as the former θ and ϕ gyros, respectively. Let δ_x and δ_y be the output angles of the x and y gyros, respectively. A resolver that turns with ψ_g resolves these two gyro outputs into two other outputs, which will be called δ_θ' and δ_ϕ':

$$\begin{aligned}\delta_\theta' &= \delta_x \cos\psi_g - \delta_y \sin\psi_g\\ \delta_\phi' &= \delta_x \sin\psi_g + \delta_y \cos\psi_g.\end{aligned} \tag{23}$$

The θ and ϕ servos are driven in accordance with (5) and (8), but with δ_θ' and δ_ϕ' replacing δ_θ and δ_ϕ, respectively.

Let ω_x and ω_y be the components of angular velocity of the third gimbal in the directions of the x-gyro input axis and the y-gyro input axis, respectively. Then, as in (3), if the gyros are ideal,

$$\begin{aligned}c\dot{\delta}_x &= H\omega_x\\ c\dot{\delta}_y &= H\omega_y.\end{aligned} \tag{24}$$

The angular velocities of the second and third gimbals differ from each other in their vertical components, but the horizontal component of one is equal to the horizontal component of the other. Therefore,

$$\begin{aligned}\omega_x &= \omega_\theta \cos\psi_g + \omega_\phi \sin\psi_g\\ \omega_y &= -\omega_\theta \sin\psi_g + \omega_\phi \cos\psi_g\end{aligned} \tag{25}$$

where ω_θ is the component of angular velocity parallel to the θ axis, and ω_ϕ is the horizontal component perpendicular to the θ axis. The components ω_θ and ω_ϕ still satisfy (2).

The vertical component of angular velocity of the third gimbal will be called ω_z.

$$\omega_z = \omega_\psi + \dot{\psi}_g = \dot{\epsilon}_\phi \sin\theta_\theta + \dot{\epsilon}_\psi \tag{26}$$

where ω_ψ is given by (2), and $\epsilon_\psi = \psi_a + \psi_g$. The heading servo drives to make ϵ_ψ zero.

From (23)-(25), for any constant ψ_g,

$$\begin{aligned}c\dot{\delta}_\theta' &= H\omega_\theta\\ c\dot{\delta}_\phi' &= H\omega_\phi.\end{aligned} \tag{27}$$

Since (27) has the same form as (3), the behavior of the system with two ideal gyros on the third gimbal is much the same as the behavior of the system with two ideal gyros on the second gimbal.

Practical Gyros

With nonideal gyros, the orientation of the output axes again affects servo performance. Let α be the angle between the x-gyro output axis and the y-gyro input axis, and let β be the angle between the y-gyro output axis and the x-gyro input axis. Then (24) is corrected to become

$$\begin{aligned}J\ddot{\delta}_x + c\dot{\delta}_x &= H\omega_x - J(\dot{\omega}_y \cos\alpha + \dot{\omega}_z \sin\alpha)\\ J\ddot{\delta}_y + c\dot{\delta}_y &= H\omega_y - J(\dot{\omega}_x \cos\beta + \dot{\omega}_z \sin\beta)\end{aligned} \tag{28}$$

just as (3) was corrected to become (13).

From (28), (23), and (25), it follows that, for any constant ψ_g,

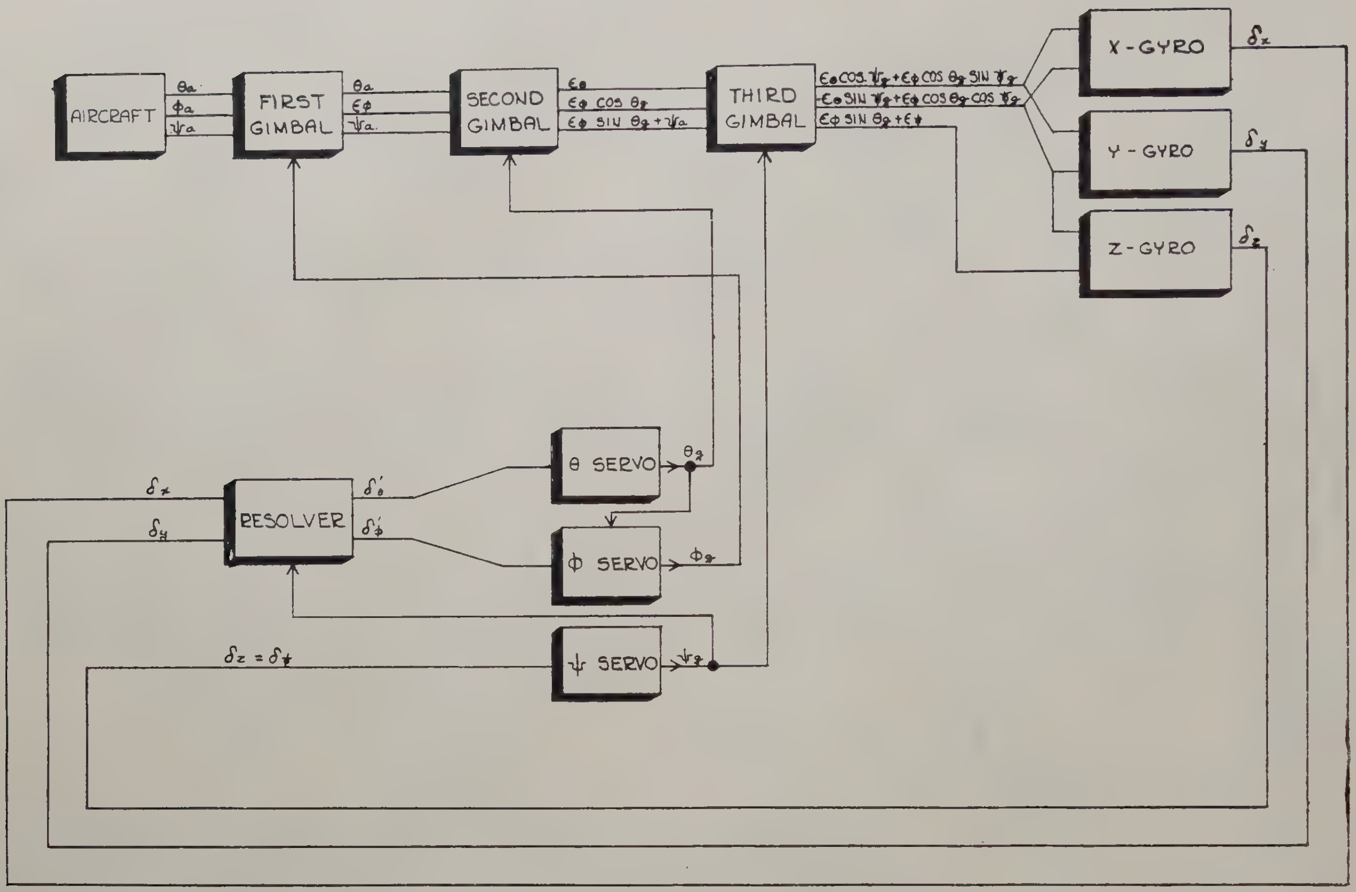


Fig. 7—Block diagram—three-gimbal system. In general, each of the three components of angular motion of the third gimbal would affect all three gyros. For the sake of clarity, however, the flow lines are shown specifically for a gyro orientation in which the three output axes are all perpendicular to the axis of the third gimbal, with the z gyro parallel to the x gyro. (This orientation is shown in Fig. 6. The x, y, z gyros are there labelled east, north, and heading, respectively.)

$$\begin{aligned} J\ddot{\delta}_\theta' + c\dot{\delta}_\theta' &= H\omega_\theta + J[\dot{\omega}_\theta \sin\psi_g \cos\psi_g(\cos\alpha + \cos\beta) \\ &\quad - \dot{\omega}_\phi(\cos^2\phi_g \cos\alpha - \sin^2\psi_g \cos\beta) \\ &\quad - \dot{\omega}_z(\cos\psi_g \sin\alpha - \sin\psi_g \sin\beta)] \\ J\ddot{\delta}_\phi' + c\dot{\delta}_\phi' &= H\omega_\phi + J[\dot{\omega}_\theta(\sin^2\psi_g \cos\alpha - \cos^2\psi_g \cos\beta) \\ &\quad - \dot{\omega}_g \sin\psi_g \cos\psi_g(\cos\alpha + \cos\beta) \\ &\quad - \dot{\omega}_z(\sin\psi_g \sin\alpha + \cos\psi_g \sin\beta)]. \end{aligned} \tag{29}$$

When $\alpha = 0°$ and $\beta = 180°$, (29) becomes

$$\begin{aligned} J\ddot{\delta}_\theta' + c\dot{\delta}_\theta' &= H\omega_\theta - J\dot{\omega}_\phi \\ J\ddot{\delta}_\phi' + c\dot{\delta}_\phi' &= H\omega_\phi + J\dot{\omega}_\theta. \end{aligned} \tag{30}$$

whether the two gyros are mounted on the second gimbal or on the third gimbal.

When α and β are both 0°, or are both 180°, the tendency of the servos to oscillate or to nutate depends upon the heading angle ψ_g.[2]

When α or β is 90° or 270°, servo stability again depends upon θ_g. For example, assume $\alpha = 90°$ and $\beta = 90°$. Then (29) becomes

$$\begin{aligned} J\ddot{\delta}_\theta' + c\dot{\delta}_\theta' &= H\omega_\theta + J\dot{\omega}_z(\sin\psi_g - \cos\psi_g) \\ J\ddot{\delta}_\phi' + c\dot{\delta}_\phi' &= H\omega_\phi - J\dot{\omega}_z(\sin\psi_g + \cos\psi_g). \end{aligned} \tag{31}$$

From (31), (2), and (26), the transforms of δ_θ' and δ_ϕ' are found to be

$$\begin{aligned} \delta_\theta' &= \frac{H\epsilon_\theta + Js(\epsilon_\phi \sin\theta_g + \epsilon_\psi)(\sin\psi_g - \cos\psi_g)}{c + Js} \\ \delta_\phi' &= \frac{[H\cos\theta_g - Js\sin\theta_g(\sin\psi_g + \cos\psi_g)]\epsilon_\phi - Js\epsilon_\psi(\sin\psi_g + \cos\psi_g)}{c + Js} \end{aligned} \tag{32}$$

By comparison of (30) with (13), it is seen that δ_θ' and δ_ϕ' satisfy the same equations as δ_θ and δ_ϕ, respectively, when $\alpha = 0°$ and $\beta = 180°$. This is also true when $\alpha = 180°$ and $\beta = 0°$. Under either of these two conditions, therefore, the performance of the θ and ϕ servos is the same

Fig. 7 illustrates the interrelationships between channels. The loop gain of the ϕ servo, G_ϕ', can be found from (32) just as (16) was found from (14):

[2] This has been verified experimentally, giving proof of the validity of this entire analysis.

$$G_\phi' = \frac{G'}{s}\frac{H}{c}\left[1 - \frac{Js}{H}\tan\theta_g(\sin\psi_g + \cos\psi_g)\right]. \qquad (33)$$

Because of the similarity between the loop gain G_ϕ' given by (33) and the loop gain G_ϕ given by (16) when $\beta = 90°$, it may be inferred that the ϕ servo will be stable only for a restricted range of θ_g. Unlike G_ϕ, G_ϕ' is a function of ψ_g; consequently, the range of θ_g is a function of ψ_g. In the previous example, where the gyros were on the second gimbal, the range of θ_g was from $-85°$ to $+65°$. For the gyros on the third gimbal, this would also be the range when $\psi_g = 0°$ or $90°$; but when $\psi_g = 180°$ or $270°$, the range would be from $-65°$ to $+85°$. Even though the range is from $-90°$ to $+90°$ when $\psi_g = \pm 45°$ or $\pm 135°$, the value of θ_g must be confined to the range from $-65°$ to $+65°$ if stability of the ϕ servo is to be insured for all values of ψ_g.

Conclusions

The final conclusion is that all gyros, whether on the second gimbal or on the third gimbal, should be mounted with their output axes perpendicular to the axis of the third gimbal; however, if 1) the aircraft is restricted in its motion so that rotations of the second gimbal are always considerably less than 90°, and 2) all the gyros are on the third gimbal, then it may be preferable to mount the gyros that control the first and second gimbals with their output axes parallel to the axis of the third gimbal. Under the specified conditions, the latter orientation will permit higher loop gains without inducing nutation.

With all the output axes perpendicular to the third gimbal axis, it is also required to pay attention to the sense of the output axis directions. For example, if the x gyro has its input axis directed to the east and its output axis directed to the north, and if the y gyro has its input axis directed to the north, then the y gyro should have its output axis directed to the west, and not to the east. (It should be noted that reversing the direction of rotation of the gyro wheel and reversing the excitation on the signal generator are together equivalent to a physical rotation of 180° of the gyro about its input axis, or to a reversal of the sense of its output axis.)

None of the above considerations is modified if the orientation of the gimbal system as a whole is altered. Although it has been assumed that the axis of rotation of the first gimbal is parallel to the longitudinal axis of the aircraft, no restriction is imposed by the assumption. The axis of the first gimbal could, for example, be parallel to the lateral axis of the aircraft, and the conclusions pertaining to gyro orientation would stand unchanged.

A Survey of Adaptive Control Systems*

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Summary—The various criteria upon which self-optimizing systems have been based are reviewed, and the operation of each type of system is discussed. A new system which is self-optimizing with respect to a measure of impulse response is described, and experimental results are presented.

Introduction

ADAPTATION—the ability of self-adjustment or self-modification in accordance with changing conditions of environment or structure—is a fundamental attribute of living organisms. It is certainly a desirable attribute for a machine. The growing interest in the problem of designing adaptive systems is evidenced by the number of papers on the subject which have appeared in recent years. A number of methods have been suggested, and it seems worthwhile to classify these according to the way that adaptive behavior is achieved. One scheme of classification is described in the next section.

In a later section a method of self-optimization based on transient response is described in detail. The design of this impulse-response adaptive system is based on three design principles:

1) A means must be provided for continuous monitoring of system performance.
2) The monitored performance must be converted to a "figure of merit"—that is, a quantitative description of the quality of performance.
3) Provision must be made for controlling system parameters by means of the figure of merit.

These principles form a useful approach to the design of adaptive systems. They are present, at least implicitly, in the various methods reviewed here.

Classification of Adaptive Systems

In order to compare the approaches which have been taken to the design of adaptive systems, we have separated them into five classes. There is, of course, a certain amount of overlapping. Some systems fall into transition zones between classes; others combine features of more than one class. The classification scheme forms, however, a convenient framework for comparison. The classes are:

1) *Passive adaptation:* Systems which achieve adaptation without system parameter changes, but rather through design for operation over wide variations in environment.
2) *Input signal adaptation:* Systems which adjust their parameters in accordance with input signal characteristics.
3) *Extremum adaptation:* Systems which self-adjust for maximum or minimum of some system variable.
4) *System-variable adaptation:* Systems which base self-adjustment on measurements of system variables.
5) *System-characteristic adaptation:* Systems which make self-adjustments based on measurement of transfer characteristics.

An alternate method of classification has been suggested by Levin [1], who divides self-optimizing systems into "input sensing," "plant sensing," and "performance-criterion sensing."

Passive Adaptation (Class 1)

The classical example of passive adaptation is feedback itself. The reduction of effects of element changes through the use of feedback is well known. There are, however, special configurations which have inherent adaptive behavior beyond that found in conventional feedback systems.

The degree of adaptive behavior in a linear feedback system can be increased by prefiltering the input signal appropriately. The design method is based on rearrangement of the configuration so that the adaptive function is apparent. This topological approach was suggested by Lang and Ham [3]. A form of their "conditional feedback" system and the equivalent prefilter system are shown in Fig. 1. The error signal ϵ will be zero only if the responses of the system G_1 and the model G_1' are the same. Thus, the system tends to have the characteristics of the model. The principle of conditional feedback has been applied to the problem of flight control of an aircraft with changing aerodynamic characteristics [2], [5], [7] with some success. A thorough discussion of systems of this type is given by Truxal [9].

In addition to the linear conditional feedback system, Lang and Ham also suggested replacing the summing device at point A in Fig. 1 by a multiplier, and that at B by a divider. This nonlinear system, which has no prefilter counterpart, opens up many possibilities for improved performance with, of course, the usual analytical difficulties associated with nonlinear systems.

Other nonlinear configurations have been proposed by Reswick [8] and Marx [6], and there seems to be no end to the possibilities here. A nonlinear system based on more conventional feedback ideas is suggested by Lewis [4]. The system, shown in Fig. 2, augments the usual error signal with rate feedback proportional to absolute value of error.

The deliberate introduction of a nonlinearity in the system represents a step toward adaptive systems of Class 4 which change parameters as a function of signals

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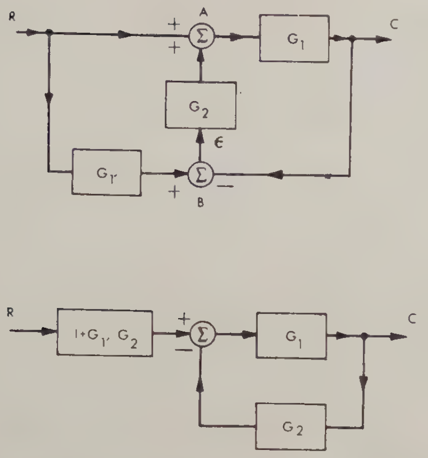


Fig. 1 —Conditional feedback and its prefilter equivalent.

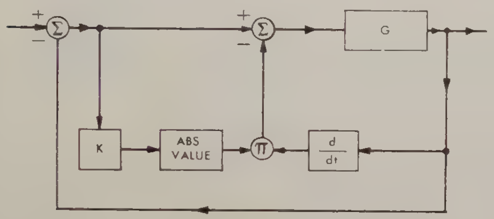


Fig. 2—Lewis' nonlinear rate feedback.

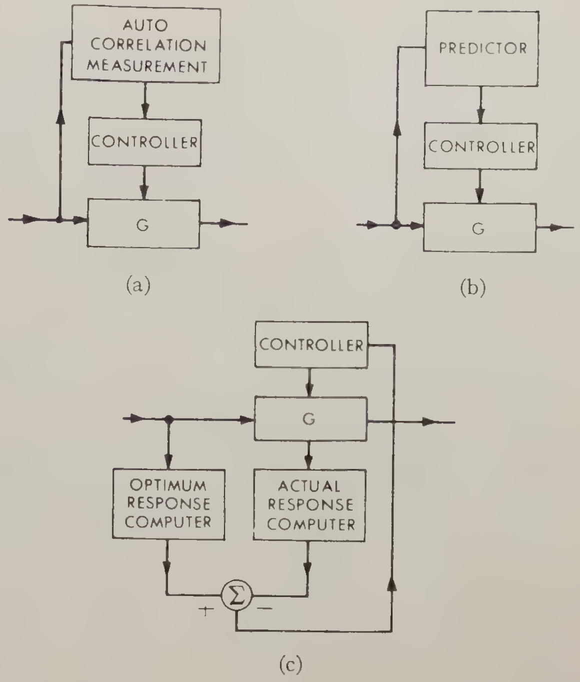


Fig. 3—Input-signal adaptive systems. (a) Keiser, (b) Drenick and Shahbender, (c) Batkov and Solodovnikov.

measured in the system. The nonlinearities in effect perform the parameter changing function in a passive way.

Input Signal Adaptation (Class 2)

Systems which adapt to changes in the characteristics of the input signal have a classical counterpart in automatic gain and frequency control. There are many input signal characteristics that can be used as the bases of adaptive system design, and several systems of this type have been proposed recently.

A system which minimizes mean-square error between signal input and system output in the presence of noise of known statistical characteristics has been described by Keiser [13]. The adjustment of system parameters is made on the basis of measurements of short-time autocorrelation of signal and noise at the input.

Burt [11] studies a system in which properties of the system transfer function, noise, and signal are such that error spectral density is a constant for minimum error.

Drenick and Shahbender [12] consider the problem of a servomechanism with polynomial plus noise input. A Zadeh-Ragazzini input predictor is used to estimate input signal characteristics which are used to adjust the system for minimum mean-square noise plus squared dynamic error. The minimization is based on detailed knowledge of the controlled elements of the system.

Batkov and Solodovnikov [10] present a method for determining the system impulse response which minimizes squared dynamic plus weighted mean-square noise error. They suggest a system which computes this optimum response and compares it with measured response, the difference being used as an error signal to be applied to a device for changing system parameters. Their proposed system differs from those in [13], [12], and [11] in that it uses a control loop to effect system parameter changes, rather than *a priori* knowledge of the controlled-system characteristics. Since their parameter adjustment is based on system characteristics, the system might be considered in Class 5. Fig. 3 compares three of these systems.

Extremum Adaptation (Class 3)

Adaptive systems designed to operate at an extremum of a system variable have received more attention in the literature than any of the other types. Therefore, we put extremum systems in a separate class, although their methods of operation are related to those in Class 4.

The principles of extremum adaptation were first presented by Draper and Li [16], [19]. Four methods for realizing adaptive behavior were proposed:

1) In Fig. 4(a) a constant-rate adjustment of the system is made so long as the measured output rate remains positive. After the performance peak is passed the output rate changes sign; when the rate reaches a preset threshold, the adjustment rate is reversed. The result is hunting operation about the optimum point.
2) In Fig. 4(b) a small sinusoidal test signal is added to the system adjustment. This signal is "modulated" by the operating characteristic of the system, the amplitude at the output being zero when the system is operating at a peak of its characteristic. The detected response to the test signal is used to control the system adjustment.
3) In Fig. 4(c) the control signal is proportional to the slope of the system characteristic, *i.e.*, the rate of change of output with respect to adjustment. A dead zone in which constant rate adjustment is applied is supplied to avoid division by zero.

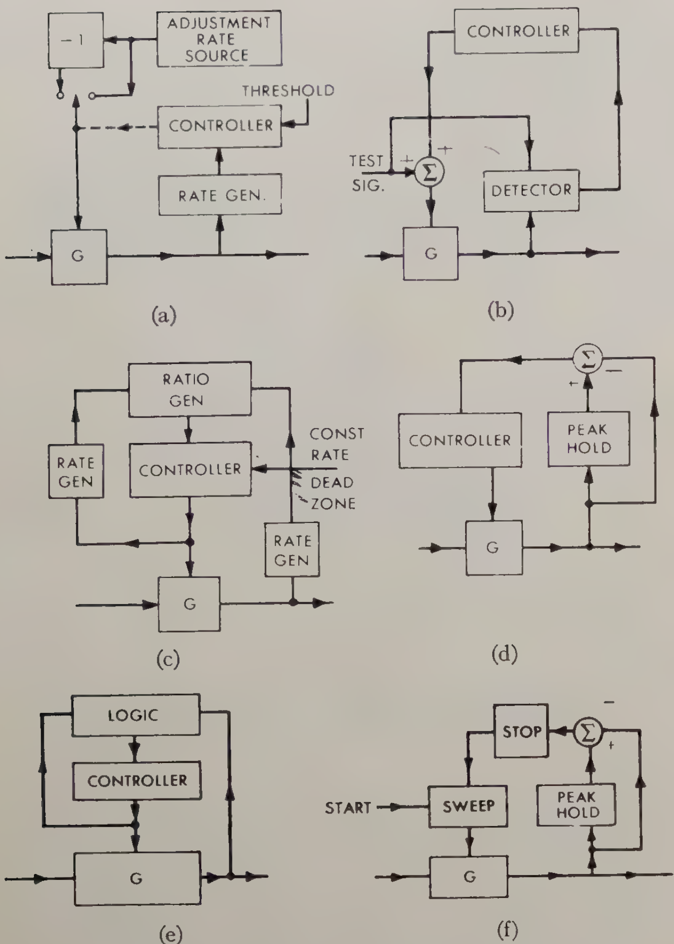


Fig. 4—Extremum adaptive systems. (a)–(d) Draper and Li, (e) Farber and Cosgriff, (f) Young.

4) In Fig. 4(d) the adjustment control signal is proportional to the difference between present output and the largest value the system has had in the past. This "peak-holding" system avoids the use of the differentiator necessary in Fig. 4(a) and 4(c) and hence is less subject to noise troubles. The use of a test signal as in Fig. 4(b), which might be objectionable, is also avoided.

An analysis of the operation of a peak-holding system using the adjustment method of Fig. 4(a)—also described in [16]—is given by Tsien and Serdengecti [23]. Draper and Li successfully tested the peak-holding system in the control of an internal combustion engine. The technique has also been applied to aircraft cruise control by Shull [22] and more recently by Genthe [18]. A commercially available process controller based on the principle is described by White [24].

Another approach to the same problem emphasizes the use of logic in the adjustment controller; *e.g.*, if both adjustment and output are increasing, no change is made; if adjustment is increasing and output is decreasing, adjustment is reversed. Farber [17] and Cosgriff [14], [15] describe systems of this type [see Fig. 4(e)].

Young [25] describes a system which scans the operating range, and then returns to the condition of maximum output [see Fig. 4(f)]. The adaptive feature does not function during normal operation as it does in the other systems previously described. Considerable work on extremum adaptive systems has appeared in the Soviet literature. Two review articles [20], [21] were published, for example, in 1957.

System-Variable Adaptation (Class 4)

Ashby's homeostat [26], [27] is a device capable of random adjustment of its own parameters whenever system-variable measurements exceed certain levels. The adjusting process continues until a stable operating condition is achieved. Although the device, according to Ashby, "does nothing more than run to a state of equilibrium,"[1] it is an example of a system which bases adjustments on the measurement of system variables.

Flügge-Lotz and Taylor [30] describe a system in which Ashby's random switching is replaced by logical switching based on simultaneous measurements of system output, error, and the derivatives of these. A second-order servo with adjustable position and rate feedback was used to verify analytical results.

A system designed to follow either a velocity or acceleration input, depending on which of two sets of parameters is in use, is described by Benner and Drenick [29]. The parameters are switched when the error signal exceeds limits determined by the desired false-alarm probability in the presence of noise. Bairnsfather [28] adjusts system parameters for minimum mean-square error, using a predetermined relationship between measured tracking error and required parameter changes. A similar method with an empirical rather than statistical criterion has been applied to aircraft flight path control by Markusen and Keeler [31]. The systems described are illustrated in Fig. 5.

The application of system-variable adaptation to humidity control is discussed by Tucker [32].

System-Characteristic Adaptation (Class 5)

System-characteristic adaptive systems base adjustments on some measure of system characteristics. One of the methods described below and the impulse-response adaptive system described in a later section use impulse response as that measure. The other method described here uses the *z*-transform transfer function of the system. There are, of course, other possibilities; *e.g.*, frequency response might be used. Goodman and Hillsley [34] discuss measurement of the moments of system impulse response as a step toward adaptive control.

Kalman [35] uses a computer to estimate the pulse transfer function of the controlled system. Adjustments are made in the controller to achieve zero error in minimum time for a step input. The characteristic measurement, which is based on observations of the controlled system input and output, breaks down when the system error becomes zero. Hence, the error never vanishes when the adaptive loop is operating. A block diagram of the system is shown in Fig. 6(a).

Seusy and Heermann [36] use the integral of the

[1] W. R. Ashby, "An Introduction to Cybernetics," John Wiley and Sons, Inc., New York, N. Y., p. 84; 1957.

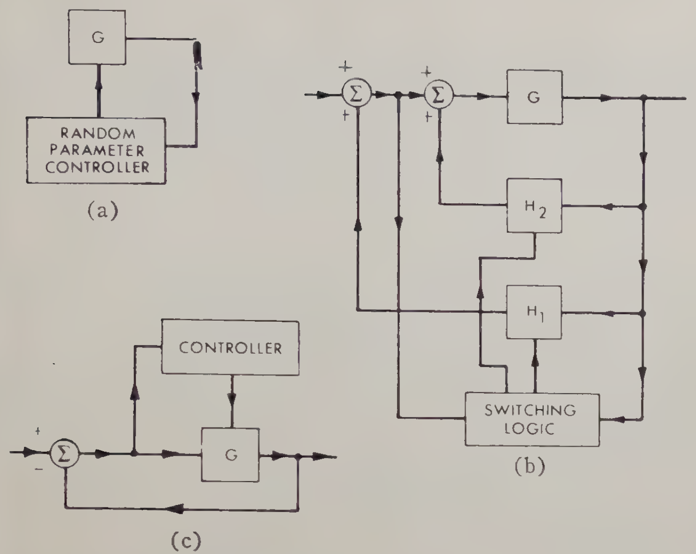


Fig. 5—System-variable adaptation. (a) Ashby, (b) Flügge-Lotz and Taylor, (c) Benner and Drenick, Bairnsfather, Markusan and Keeler.

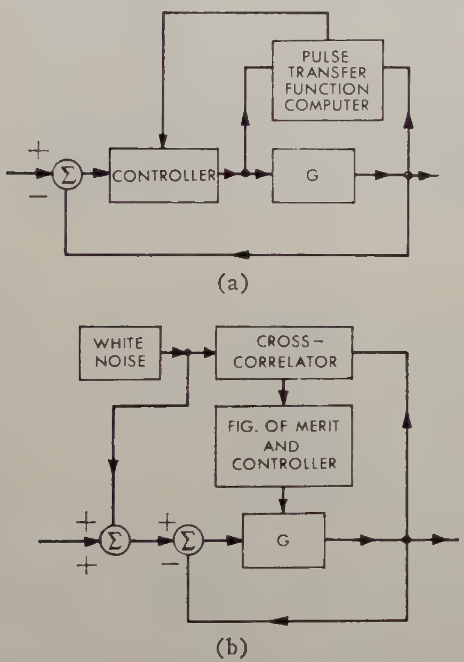


Fig. 6—System-characteristic adaptation. (a) Kalman, (b) Anderson, *et al.*

time-multiplied absolute value of error (ITAE) as the basis for system adjustment. The ITAE has a minimum for second and third-order systems near optimum damping. The system is subjected to an initial condition, the ITAE is obtained and stored, and the cycle is repeated with a slight reduction in parameter values. When the minimum is reached, the stored values do not change in successive cycles, and adjustment stops.

Anderson *et al.* [33] measure system impulse response by the cross-correlation function of input and output when white noise is applied. A figure of merit is derived from the impulse response by forming a weighted sum of positive and negative areas:

$$F_M = A_+ + R_0 A_-.$$

The behavior of F_M is shown in Fig. 7. The function can

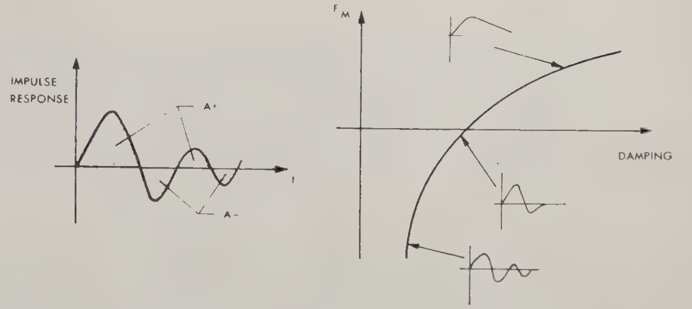


Fig. 7—Figure of merit.

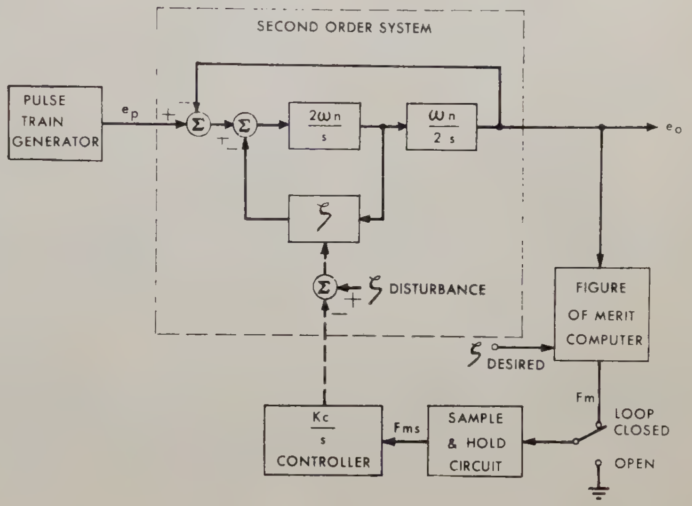


Fig. 8—Impulse excited optimizing loop.

be made to change sign at the desired system damping by proper choice of R_0, and hence is suitable as an adjustment control signal. The impulse-response adaptive system described in this paper uses the same general approach, but employs periodic impulse excitation in place of the noise measurement.

Tabulation of Adaptive Systems

In Table I (next page) we have listed several systems from each class and summarized the features in each.

An Impulse-Response Adaptive System

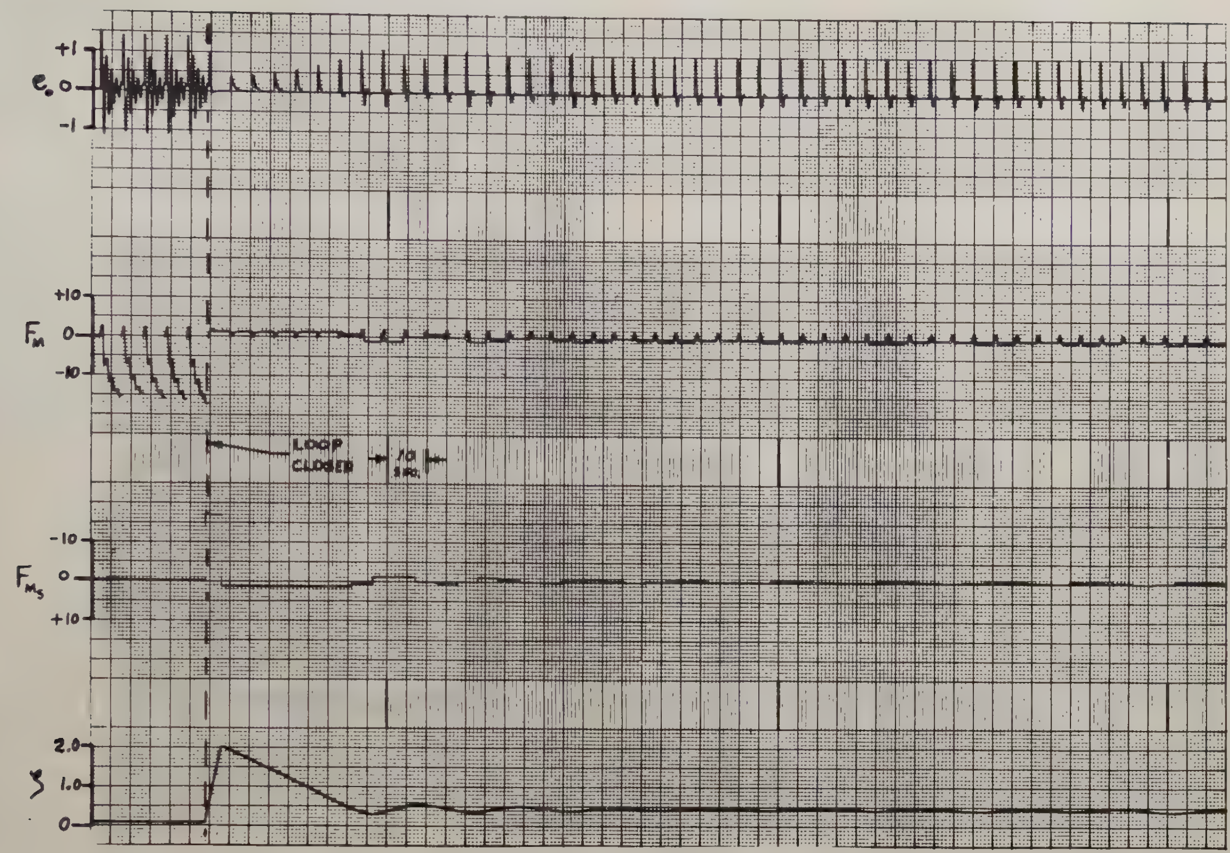
To illustrate the principle of impulse-response adaptation, an impulse-excited adaptive system was studied on an analog computer by the authors. The system, shown in Fig. 8, consisted of a second-order system with damping ratio ζ controlled by the adaptive loop. The adjustment was obtained from the area-ratio figure of merit described above. Unlike the system described in [33] the impulse response was obtained directly by exciting the system with periodic impulses. Fig. 9(a) and 9(b) shows records of the recovery of the system from initial under and over-damped conditions.

The figure of merit F_M was obtained by integrating the sum of positive and weighted negative parts of the impulse response e_0 each time an impulse was applied. The final value F_{M_s} was sampled and held in synchronism with the applied impulses.

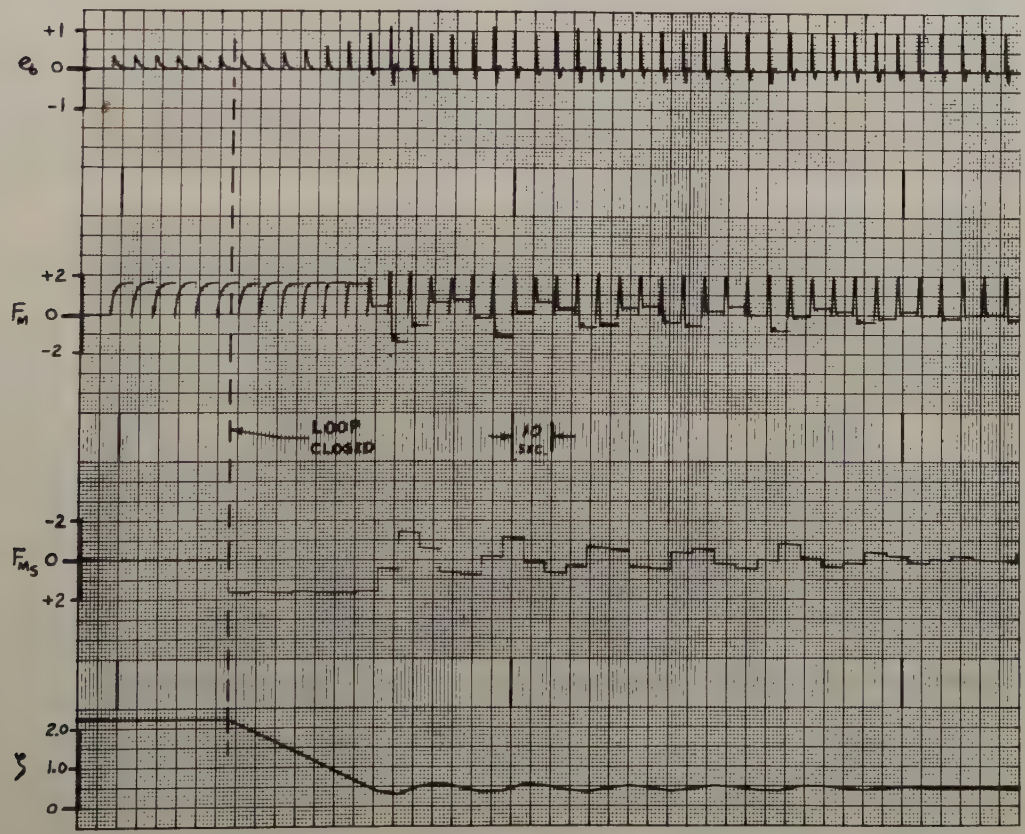
Since the impulses may cause undesirable interference, systems of this type can be applied only where

TABLE I
CHARACTERISTICS OF ADAPTIVE SYSTEMS

Author	Function of system	Adapts to what?	Means of adaptation	Criterion for adjustment
Class 1—Passive adaptation				
Lang and Ham [3]	Servomechanism	Input and disturbance changes	Conditional feedback	Model
Lewis [4]	Servomechanism	Input changes	Rate feedback proportional to error signal	Low damping for large errors and vice versa
Marx [6]	Flight control system	Changes in system	Multiplier-divider provides zero error for acceleration commands	Zero error
Rath [7]	Flight control system	Changes in system parameters	Linear conditional feedback	Response specified by model
Reswick [8]	Process control	Changes in parameters	Inverse model changed by measurements of controlled process to correspond to latter	Minimize effects of disturbances
Class 2—Input-signal adaptation				
Batkov and Solodovnikov [10]	Follows signal in presence of noise	Changes in input and environment	Changes system to conform with optimum as determined from input characteristics	Conformity with optimum impulse response as determined; minimizes square of weighted noise and dynamic error $(\epsilon_d^2+\lambda(t)\epsilon_{ms}^2)$
Drenick and Shahbender [12]	Servomechanism with polynomial input	Changes in input	Gain adjustment in accordance with predicted input signal (Zadeh-Ragazzini predictor)	Minimum squared dynamic plus noise error $(\epsilon_d^2+\epsilon_{ms}^2)$; gain dependence determined from knowledge of controlled system
Class 3—Extremum adaptation				
Cosgriff and Emerling [14], [15]	Maximizes system output	Changes in system	Logic used to drive system to maximum; stable equilibrium at maximum	Maximum output
Draper and Li [16]	Internal combustion engine	Changes in environment	Peak-holding optimalizing control	Maximum load torque at constant speed
Young [25]	Transmitter tuning	Changes in exciter frequency	Entire amplifier range scanned; system returns to point of maximum output	Maximum system output
Class 4—System-variable adaptation				
Ashby [26], [27]	Demonstrates "purposeful behavior"	Changes in system parameters and structure	Changes system parameters	Stable equilibrium
Bairnsfather [28]	Servo with Gaussian input	Changes in input statistics and fixed element parameter changes	Gain adjustment based on level of system error	Minimum mean-square error
Benner and Drenick [29]	Follows sampled input; input assumed constant speed or acceleration	Changes in input from constant speed to acceleration or vice versa	Switches between preselected system parameters	Recognition of input changes in presence of noise with desired false-alarm probability
Markusen and Keeler [31]	Flight path control	Noise level in received signal	Variation of error limit and loop gain with noise level in error signal	Experimentally determined optimum as function of noise level
Class 5—System-characteristic adaptation				
Anderson *et al.* [33]	Servomechanism	Changes in system parameters due to environmental changes	Changes in system parameters in accordance with area-ratio figure of merit	"Optimum" impulse response
Kalman [35]	Servo with step input	Changes in characteristics of process being controlled	Changes in digital controller parameters; estimates process pulse-transfer function and computes control parameters	Zero error with step input in minimum time
Seusy and Heermann [36]	Servomechanism	Changes in system parameters due to environment	Parameters changed until ITAE remains constant	ITAE minimum



(a)



(b)

Fig. 9—(a) Impulse excited system initially underdamped, (b) impulse excited system initially overdamped.

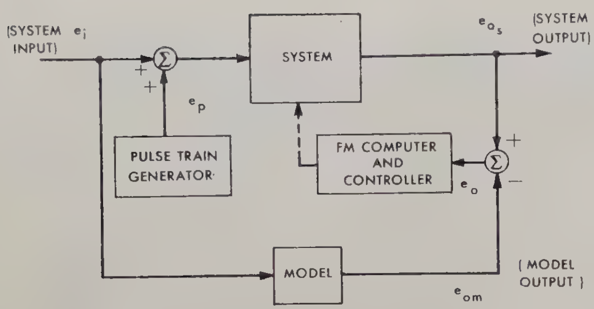


Fig. 10—Impulse excited model system.

such interference is not a problem. A means for reducing the effects of command inputs on the adaptive loop is shown in Fig. 10. This system combines a model of the desired system with the adaptive loop to eliminate command response from the figure of merit computer.

Conclusions

The survey undertaken here indicates the variety of approaches that have been taken to the problem of design of adaptive systems. Although we have classified them according to the method used in achieving adaptive behavior, we might have grouped them by function. There are systems to achieve optimum steady-state operation, others designed for transient performance, and still others for some special type of response or operation. We make no attempt to say which system is best—that depends on how well the system performs its task.

It is well to try to discover some underlying unity in this broad field. We think that in time this may come about through the solution of a problem which is as yet unsolved—the problem of setting basic limitations on adaptive behavior. In order for a system to adapt, use must be made of knowledge about present system performance. This implies disturbing the system and waiting for the response. How large a disturbance is needed, and how long must we wait? How can we minimize *a priori* knowledge required about the system? How can this minimum be determined? In answers to these questions may lie fundamental principles of adaptive control.

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[8] J. B. Reswick, "Disturbance-response feedback—a new control concept," *Trans. ASME*, vol. 78, pp. 153–162; January, 1956.
[9] J. G. Truxal, "Modern network theory and its applications to feedback control," *Proc. Conf. on Systems Eng.*, Purdue Univ., Lafayette, Ind., pp. 79–104; July, 1955.

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[12] F. F. Drenick and R. A. Shahbender, "Adaptive servomechanisms," AIEE Trans. Paper No. 57-388; July, 1957.
[13] B. E. Keiser, "The linear input-controlled variable-pass network," IRE Trans. on Information Theory, vol. IT-1, pp. 34–39; March, 1955.

Class 3—Extremum Adaptation

[14] R. Cosgriff, "Servos that use logic can optimize," *Control Eng.*, vol. 2, pp. 133–135; September, 1955.
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[16] C. S. Draper and Y. T. Li, "Principles of optimalizing control systems and an application to the internal combustion engine," *ASME publication;* September, 1951.
[17] B. Farber, "Computer circuit finds peaks automatically," *Control Eng.*, vol. 1, pp. 70–75; October, 1954.
[18] W. K. Genthe, "Optimalizing control-design of a fully automatic cruise control system for turbojet aircraft," 1957 WESCON Convention Record, pt. 4, pp. 47–57.
[19] Y. T. Li, "Optimalizing system for process control," *Instruments*, vol. 25, pp. 72–77, 190, 324; 1952.
[20] I. S. Morosanov, "Optimalizing control methods," *Avtomat. i Telemechan.*, vol. 18, pp. 1029–1044; November, 1957 (in Russian).
[21] Y. I. Ostrovsky, "Extremum regulation," *Avtomat. i Telemechan.*, vol. 18, pp. 852–858; September, 1957 (in Russian).
[22] J. R. Shull, "An automatic cruise control computer for long range aircraft," IRE Trans. on Electronic Computers, no. PGEC-1, pp. 47–51; December, 1952.
[23] H. S. Tsien and S. Serdengecti, "Analysis of peak-holding optimalizing control," *J. Aeronaut. Sci.*, vol. 22, pp. 561–570; August, 1955.
[24] B. White, "The quarie optimal controller," *Instr. and Automation*, vol. 29, pp. 2212–2216; November, 1956.
[25] N. H. Young, "An automatic control system with provision for scanning and memory," *Trans. AIEE*, vol. 72, pt. 1, pp. 392–395; September, 1953.

Class 4—System-Variable Adaptation

[26] W. R. Ashby, "An Introduction to Cybernetics," John Wiley and Sons, Inc., New York, N. Y.; 1957.
[27] W. R. Ashby, "Design for a Brain," John Wiley and Sons, Inc., New York, N. Y.; 1952.
[28] R. R. Bairnsfather, "A self-adjusting control system," thesis for M.S. degree, M.I.T., Cambridge, Mass.; June, 1956.
[29] A. H. Benner and R. Drenick, "An adaptive servo system," 1955 IRE Convention Record, pt. 4, pp. 8–14.
[30] I. Flügge-Lotz and C. F. Taylor, "Investigation of a Nonlinear Control System," NACA Tech. Note 3826; April, 1957.
[31] D. L. Markusen and R. J. Keeler, "A noise adaptive flight path control system," *Proc. AIEE Second Feedback Control Systems Conf.*, pp. 115–122; April, 1954.
[32] G. K. Tucker, "An adaptive humidity control system," ASME Paper No. 58-IRD-1; April, 1958.

Class 5—System-Characteristic Adaptation

[33] G. W. Anderson, J. A. Aseltine, A. R. Mancini, and C. W. Sarture, "A self-adjusting system for optimum dynamic performance," 1958 IRE National Convention Record, pt. 4, pp. 182–190.
[34] T. P. Goodman and R. H. Hillsley, "Continuous measurement of characteristics of systems with random inputs: a step toward self-optimizing control," ASME Paper No. 58-IRD-5; April, 1958.
[35] R. E. Kalman, "Design of a self-optimizing control system," ASME Paper No. 57-IRD-12; May, 1957.
[36] F. E. Seusy and C. R. Heermann, "Principles of Optimizing Control Applied to Analog Computer Problems," USAF Inst. of Tech., Rep. GACA 54-9; March, 1954.

Stability of Forced Oscillations in Nonlinear Feedback Systems*

ZE'EV BONENN†

Summary—It has been known for a considerable time that nonlinear systems may exhibit multivalued response under external periodic excitation. The stability of these forced oscillations has been extensively investigated for second-order systems and a general criterion for their stability was found. Stability of higher order systems was investigated by means of the incremental describing function. This function must be calculated for every case of interest. In this paper the general criterion formerly derived for second-order systems is extended to higher order systems. Thus it is not necessary to make a special stability investigation in every case.

Introduction

The phenomenon of multivalued response in a nonlinear system excited by a periodic input is well known.[1–3] Consider, for example, the system given in Fig. 1, where G is a linear transfer function preceded by a nonlinear element with the describing function N. For a sinusoidal input we may write

$$E \cdot \{1 + N(E) \cdot G\} = R. \tag{1}$$

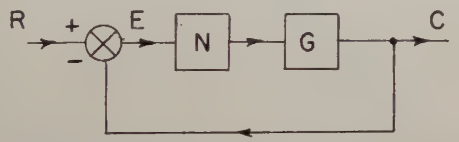

Fig. 1—Block diagram representing a nonlinear system.

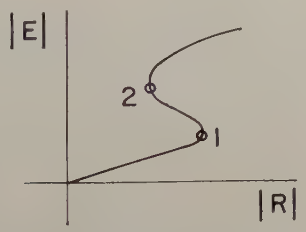

Fig. 2—Error magnitude as a function of input magnitude in a nonlinear system.

The explicit solution $E=f(R)$ may be found by graphical methods.[4,5] In some cases E is a multivalued function of R for a certain range of R (Fig. 2). In this case the question naturally arises as to which values of E actually appear in the steady state. It has been proved for second-order systems that the values of E for which

$$\frac{d|E|}{d|R|} < 0$$

are unstable.[3] Thus E in region 1–2 of Fig. 2 cannot exist in the steady state. For higher order systems the same criterion has been frequently used, justified by physical reasoning but without exact proof. The purpose of this paper is to prove the validity of this criterion for higher order systems.

The Incremental Describing Function[6]

The stability of the forced oscillations of higher order systems may be investigated by means of the incremental describing function. Suppose the steady state input to the nonlinear element is $E=a\cos(wt+\phi)$. Add a small signal $b\cos wt$,[7] $b \ll a$. The input is now

$$E = a\cos(wt+\phi) + b\cos wt. \tag{2}$$

Its output will contain components due to both inputs. The incremental describing function of the nonlinear element is defined as the ratio between the components appearing in its output due to b divided by b.

The incremental describing function is given by

$$N_i(a, \phi) = N(a) + aN'(a) \cdot \cos\phi(\cos\phi + i\sin\phi).^8 \tag{3}$$

It is a function of two variables: a, the amplitude of the steady-state input, and ϕ, an arbitrary phase angle running from 0 to 2π.

If the steady-state response a is stable, then the incremental signal b must diminish with time. This is obtained by the plot of the incremental Nyquist diagram

$$1 + N_i(a, \phi) \cdot G = 0 \tag{4}$$

or

$$G = -\frac{1}{N_i(a, \phi)}. \tag{5}$$

If the G locus encloses the family of curves for

$$-\frac{1}{N_i(a, \phi)}$$

for a certain range of amplitudes and frequencies, the

* Manuscript received by the PGAC, February 18, 1958.
† Senior Res. Eng., Sci. Dept., Ministry of Defense, Israel.

1 N. Minorsky, "Introduction to Nonlinear Mechanics," J. W. Edwards, Ann Arbor, Mich.; 1947.

2 C. Hayashi, "Forced Oscillations in Nonlinear Systems," C. E. Tuttle Co.; 1953.

3 K. Klotter and E. Pinney, "A comprehensive stability criterion for forced vibrations in nonlinear systems," *J. Appl. Mech.*, vol. 20, pp. 9–12; March, 1953.

4 J. C. West and P. N. Nikiforuk, "The frequency response of a servomechanism designed for optimum transient response," *Trans. AIEE Applications and Ind.*, pp. 234–239; September, 1956.

5 J. C. Lozier, "Steady state approach to saturable servomechanisms," IRE Trans. on Automatic Control, no. PGAC-1, pp. 19–39; May, 1956.

6 J. C. West, J. L. Douce, and R. K. Livesley, "The dual input describing function and its use in the analysis of nonlinear feedback systems," *Proc. IEE*, vol. 103, pt. B, pp. 463–474; July, 1956.

7 The stability of harmonic or subharmonic responses, which may be treated similarly, is not considered in this paper.

8 See Appendix.

incremental signal will increase and the steady-state values in this range are unstable.

The determination of the steady-state response requires the following steps:

1) calculation of $E=f(R)$
2) calculation of $N_i(a, \phi)$
3) plot of $G=-(1/N_i)$ to determine the stability of the values obtained in the first step.

Extension of the Stability Criterion to Higher Order Systems

Assume an input R_0 resulting in an error Eo. Then for R_0+dR we shall get $Eo+dE$. The output of the nonlinear element becomes:

$$\begin{aligned}&(Eo + dE)N(Eo + dE)\\&= (Eo + dE)\{N(Eo) + dEN'(Eo) \cdots\}\\&= EoN(Eo) + dE\{N(Eo) + EoN'(Eo) + dE^2N'(Eo)\}. \quad (6)\end{aligned}$$

Dropping the second-order term and noting that

$$N(Eo) + EoN'(Eo) \equiv N_i(Eo) \text{ by definition,} \quad (7)$$

we obtain

$$(Eo + dE)N(Eo + dE) = EoN(Eo) + dEN_i(Eo). \quad (8)$$

The input error relation for an input $Ro+dR$ is

$$Eo\{1 + N(Eo)\cdot G\} + dE\{1 + N_i(Eo)\cdot G\} = Ro + dR. \quad (9)$$

After cancelling the steady-state terms, we obtain

$$\frac{dE}{dR} = \frac{1}{1 + N_iG}. \quad 10)$$

When the denominator is zero (stability limit)

$$\frac{dE}{dR} = \infty \text{ (points 1 and 2, Fig. 2).} \quad (11)$$

It therefore follows that for

$$\frac{d|E|}{d|R|} < 0$$

the steady-state solution is unstable. Thus the criterion found for second-order systems holds for higher order systems too. It is therefore only necessary to calculate $E=f(R)$; the stability is given by the slope of the curve.

Conclusion

A general stability criterion for forced oscillations in nonlinear systems with one nonlinear element has been proven. The stability is given by the slope of the curve of the steady-state solution. The validity of the criterion depends on the validity of the describing function method in the system under investigation.

Appendix

Derivation of the Incremental Describing Function

For an odd, single-valued nonlinear element (Fig. 3) with a sinusoidal input $E=a \cos (wt+\phi)$, the fundamental component of the output is given by

$$O(a) = \frac{1}{\pi}\int_{-\pi}^{+\pi} f(a \cos wt) \cos wtdwt$$

and the describing function is

$$N(a) = \frac{O(a)}{a}. \quad (12)$$

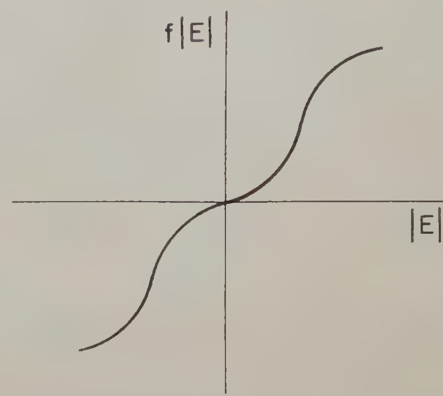

Fig. 3—Representation of an odd, single-valued nonlinear element.

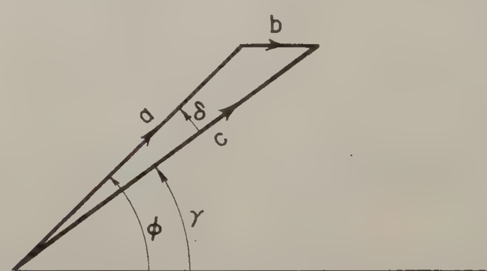

Fig. 4—Graphical representation of the input.

Add a small signal $b \cos wt$, $(b/a)\ll 1$. The input may be represented by $ce^{i\gamma}$ (Fig. 4) where

$$c = \sqrt{a^2 + b^2 + 2ab \cos \phi} \approx a + b \cos \phi \quad (13)$$

and the output is

$$\begin{aligned}O(c) &= e^{i\gamma}\cdot O(a + b \cos \phi)\\&\cong e^{i\gamma}\cdot \left\{O(a) + b \cos \phi \frac{dO}{da}\right\}. \quad (14)\end{aligned}$$

The phase angle of the output is

$$\gamma = \phi - \delta. \quad (15)$$

δ is a small angle and therefore

$$\sin \delta \approx \frac{b}{a}\cdot \sin \phi; \qquad \cos \delta \approx 1. \quad (16)$$

Thus

$$\begin{aligned}e^{i\gamma} &= e^{i\phi}\cdot e^{-i\delta} \approx (\cos \phi + i \sin \phi)\left(1 - i\frac{b}{a} \sin \phi\right)\\&= \cos \phi + \frac{b}{a} \sin^2 \phi + i\left(\sin \phi - \frac{b}{a} \sin \phi \cos \phi\right). \quad (17)\end{aligned}$$

Substituting for $e^{i\gamma}$ in (14) we obtain for the output

$$O(c) = \left[\cos\phi + \frac{b}{a}\sin^2\phi + i\left(\sin\phi - \frac{b}{a}\sin\phi\cos\phi\right)\right]$$

$$\cdot\left[O(a) + b\cos\phi\frac{dO}{da}\right]$$

$$= O(a)[\cos\phi + i\sin\phi] +$$

$$\frac{O(a)}{a}\cdot[b\sin^2\phi - ib\sin\phi\cos\phi]$$

$$+ b\cos\phi\frac{dO}{da}[\cos\phi + i\sin\phi] + b^2[\cdots]. \quad (18)$$

The component due to b is

$$O(b) = b\left\{N(a)[\sin^2\phi - i\sin\phi\cos\phi]\right.$$

$$\left. + \frac{dO}{da}[\cos^2\phi + i\sin\phi\cos\phi]\right\}. \quad (19)$$

Noting that

$$\frac{dO}{da} = O'(a) = N(a) + aN'(a) \quad (20)$$

we finally obtain

$$N_i(a, \phi) = \frac{O(b)}{b}$$

$$= N(a) + aN'(a)\cdot\cos\phi(\cos\phi + i\sin\phi). \quad (21)$$

When N_i is drawn as a function of ϕ with a as parameter we obtain a family of circles (Fig. 5). This formula is correct only when $N'(a)$ exists.

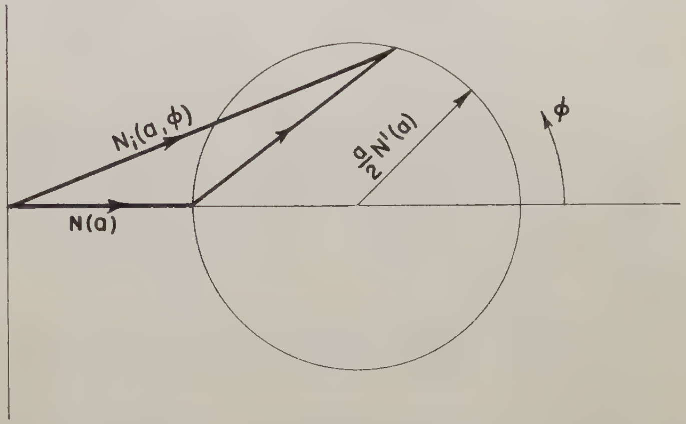

Fig. 5—N_i as a function of ϕ and $\bar{a}$.

A Selective Bibliography on Sampled Data Systems*

PETER R. STROMER†

After submitting this bibliography, the author discovered that of Freeman and Lowenschuss on the same subject, published in the PGAC TRANSACTIONS.[1] A review indicates that both had the same purpose—to simplify the task of searching the literature. Accordingly, this bibliography can be considered a supplement. The brief annotations furnished may enhance further the common goal.

Summary—**References have been compiled on the synthesis and analysis of sampling servo systems as differentiated from continuous servo systems. No attempt has been made to cover material before 1955 except where particular references have been cited by various authors repeatedly, indicating "classic" references in this comparatively new area of feedback control system literature. Items are listed alphabetically by author.**

[1] W. H. Ball, "Study of sampled-data feedback control systems with two sampling periods, one a multiple of the other," Ph.D. dissertation, Cornell Univ., Ithaca, N. Y., 99 pp.; 1957. Abstract published in *Dissertation Abstracts*, pp. 2954–2955; December, 1957.

Z-transform method of analysis is found superior to method whereby sampled-data system is approximated by an equivalent continuous-data system. *Z*-transform techniques are used to study a third-order servomechanism.

[2] R. H. Barker, "The pulse transfer function and its application to sampling servo systems," *Proc. IEE*, vol. 99, pt. 4, pp. 302–317; December, 1952.

Monograph describes the sequence transform method. The pulse transfer function relates a sequence of samples at the output of a system to the sequence producing it. Identical paper appears in [40].

[3] A. R. Bergen and J. R. Ragazzini, "Sampled-data processing techniques for feedback control systems," *AIEE Trans.*, vol. 73, pt. 2, pp. 236–247; November, 1954.

Theory, design, limitations, and applications of sampled-data systems. Time-domain synthesis.

[4] J. E. Bertram and G. Franklin, "Sampled-data feedback improves system response," *Control Eng.*, vol. 2, pp. 107–111; September, 1955.

Typical system employing digital controller. Cursory mathematical treatment, *Z*-transform omitted.

[5] J. E. Bertram, "Factors in the design of digital controllers for sampled-data control systems," *AIEE Trans.*, vol. 75, pp. 151–159; July, 1956.

Extension of time domain synthesis method proposed by Bergen and Ragazzini in [3]. This paper, which is mainly concerned with the response at the sampling instants, indicates that if a system is to follow both step and ramp inputs a compromise between settling time and overshoot at the sampling instants should be made.

[6] A. B. Bishop, "A model for optimum control of stochastic sampled-data systems," *Operations Res.*, vol. 5, pp. 546–550; August, 1957.

Principal development concerns a simple proportional control system with fixed proportionality constant.

[7] M. Blum, "An extension of the minimum mean square prediction-theory for sampled input signals," IRE TRANS. ON INFORMATION THEORY, vol. IT-2, pp. 176–184; September, 1956.

Filter theory. Method developed for finding the ordinates of a digital filter.

[8] S. S. L. Chang, "Statistical design theory for strictly digital sampled-data systems," *Commun. and Electronics*, no. 34, pp. 702–709; January, 1958.

Covers both finite and infinite memory versions of optimum filter and optimum control system synthesis. Provides definitions of correlation sequences and correlation series for sampled signals. Optimization of a strictly digital system based on the mean-square-error criteria.

[9] R. K. Cheng, "Sampled-data representation of nonlinear systems," Ph.D. dissertation, Purdue Univ., Lafayette, Ind., 145 pp.; 1957. Abstract published in *Dissertation Abstracts*, p. 1973; September, 1957.

Results provide a useful recursion formula for time-domain synthesis of linear and nonlinear networks whose input-output relations may be approximated in sampled-data form. Stability of nonlinear sampled-data systems is examined for first and second order systems.

[10] H. Chestnut, A. Dabul, and D. Leiby, "Analog Computer Study of Sampled-Data Systems," Gen. Elec. Tech. Info. Ser., Rep. 57GL351; November 15, 1957.

Analysis of second-order sampled-data servo system applying most of the known analytical techniques as well as analog computer simulation for obtaining transient performance. Paper presented at Conference on Computers in Control Systems, Atlantic City, N. J., October 16–18, 1957. A General Electric Company Technical Information Series Report available from General Electric Co., General Eng. Lab., Schenectady, N. Y.

A. Dabul. See H. A. Chestnut.

[11] S. Demczynski, "The design of sampling servomechanisms," *Elec. Energy*, vol. 1, pp. 410–414; September, 1957; pp. 446–450; October, 1957.

Limited to linear sampling servos. Part 1 describes salient features of sampling servos, reviews impulse response techniques, Laplace transform applications. Part 2 reviews basic theory of *Z* transforms and compensating techniques involving the use of digital computers.

[12] O. I. Elgerd, "A Study of the Transient Behavior and Stability Characteristics of Sampled-Data Systems," Gen. Elec. Tech. Info. Ser., Rep. 57ICS401; August 8, 1957.

Characteristics investigated by simulation on a differential analyzer. Systems classified on a root-locus basis, depending on the location of closed-loop poles and zeros. A General Electric Company Technical Information Series Report available from Gen. Elec. Co., Industry Control Dept., Roanoke, Va.

[13] G. Farmanfarma, "Analysis of linear sampled-data systems with finite pulse width: openloop," *Applications and Ind.*, no. 28, pp. 808–810; January, 1957.

P-transform method introduced with *P* denoting pulse width. With the aid of given tables it is possible to compute the output of a finite pulsed system as a continuous function of time.

[14] G. F. Franklin, "Linear filtering of sampled-data," 1955 IRE CONVENTION RECORD, pt. 4, pp. 119–128.

Application of least squares filtering theory to situations where a time stationary random message and additive noise are sampled before being filtered. Applicable to pulsed radar or data link.

G. F. Franklin. See J. E. Bertram.

[15] Franklin Institute, "A new sampled-data analogue computer," *Automatic Control*, vol. 8, p. 44; December, 1957.

Nuclear Engineering Division of the Franklin Institute Labs., Philadelphia, Pa., is constructing a sampled-data analog computer with a standard stepping relay as the key computing element. Relays reduce the number of operational amplifiers, servomultipliers and electronic multipliers which are required to represent partial differential equations.

[16] B. Friedland, "A technique for the analysis of time-varying sampled-data systems," *Applications and Ind.*, no. 28, pp. 407–414; January, 1957.

Applicable to systems where the transmission matrix of each component of the system is known. Can be used to calculate improvement of feedback system performance only by the addition of a time-varying amplifier.

* Manuscript received by the PGAC, March 28, 1958.
† General Electric Co., Schenectady, N. Y.
[1] H. Freeman and O. Lowenschuss, "Bibliography of sampled-data control systems and *Z*-transform applications," IRE TRANS. ON AUTOMATIC CONTROL, no. PGAC-4, pp. 28–30; March, 1958.

[17] D. J. Gimpel, "Sampled-data systems," *Control Eng.*, vol. 4, pp. 99–106; February, 1957.

Brief, yet comprehensive, survey of the sampled-data field. System properties discussed as well as the more important methods of predicting system performance. Good bibliography.

[18] J. D. Graham, "Determination of the effects of finite-time sampling and of nonlinear elements on the performance of sampled-data system," Ph.D. dissertation, Univ. of Wisconsin, Madison, Wis., 108 pp.; 1957. Abstract published in *Dissertation Abstracts*, p. 2549; November, 1957.

Describing function method of analysis extended to enable determination of limit cycle phenomena in both zero-time and finite-time systems. Accuracy of analysis confirmed by simulation of several sampled-data systems on the Wisconsin-Philbrick Analog Computer.

[19] G. W. Johnson, D. P. Lindorff, and C. G. A. Nordling, "Extension of continuous data system design techniques to sampled data control systems," *AIEE Trans.*, vol. 74, pt. 2, pp. 252–263; September, 1955.

Z-transform concepts extended to show feasibility of applying root-locus plots, asymptotic frequency plots, and Routh's criterion to sampled-data systems.

[20] E. I. Jury, "Analysis and synthesis of sampled-data control systems," *AIEE Trans.*, vol. 73, pt. 1, pp. 332–346; September, 1954.

Z-transform method used and clarified. An extended list of *Z*-transforms and their inverse is presented in tabular form.

[21] E. I. Jury, "Correlation between root-locus and transient response of sampled-data control systems," *AIEE Trans.*, vol. 74, pt. 2, pp. 427–435; January, 1956.

Author correlates frequency locus of the transfer function and the root-locus in the *Z* domain. Demonstrates that synthesis of sampled-data control systems can be performed by phase-angle loci. Choice between root-locus and frequency locus method of synthesis dependent on the convenience in plotting the root-locus. Root-locus recommended for those systems using pulsed networks or digital computers; frequency locus preferable for systems using networks in the continuous element.

[22] E. I. Jury, "Synthesis and critical study of sampled-data control systems," *Applications and Ind.*, no. 25, pp. 141–151; July, 1956.

Z-transform method recommended for analysis and synthesis of sampled-data systems. The method yields information only on the response of the system at sampling instants. However, modifications described by the author result in information on the system's behavior between sampling instants.

[23] E. I. Jury, "Hidden oscillations in sampled-data control systems," *Applications and Ind.*, no. 28, pp. 391–395; January, 1957.

Oscillations impose certain limitations on the *Z*-transform method and other synthesis methods. It is shown that these oscillations appear in the response because of certain forms of the open-loop transfer function. Modified *Z*-transform method presented to detect the presence of these oscillations and to indicate their values.

[24] E. I. Jury and W. Schroeder, "Discrete compensation of sampled-data and continuous control systems," *Applications and Ind.*, no. 28, pp. 317–325; January, 1957.

Compensation method reduces to zero any overshoot or error after a prescribed finite time. Response is applicable to step or ramp inputs and could be extended to acceleration or higher order inputs. Modified *Z*-transform method.

[25] E. I. Jury, "Additions to the modified *Z*-transform method," 1957 WESCON CONVENTION RECORD, pt. 4, pp. 136–156.

Author amplifies and expands his earlier work on modified *Z*-transform techniques. Tables of various sampled-data system configurations are given to indicate the transfer function of such configurations. An extensive table of the modified *Z*-transform is included.

[26] D. R. Katt, "Conditional feedback systems applied to stabilizing a missile in pitch attitude," 1957 WESCON CONVENTION RECORD, pt. 4, pp. 171–175.

Sampled-data technique applied to a conventional aircraft attitude autopilot in a flight control system.

[27] R. C. Klein, "Analog simulation of sampled-data systems," IRE TRANS. ON TELEMETRY AND REMOTE CONTROL, vol. TRC-1, pp. 2–7; May, 1955.

Sampled-data systems are considered which use discontinuous amplitude modulated data as well as digital processing units. Simulation determined on basis of theoretical considerations. Relay circuitry presented for effective use in analog simulation of sampled-data systems.

[28] C. H. Knapp, E. Shapiro, and R. A. Thorpe, "An error-sampled sweep-position control system," *IBM J. Res. and Dev.*, vol. 2, pp. 14–35; January, 1958.

Forward-path digital compensation used to achieve high accuracy and fast response. Description of the design and operation of a transistor system prototype developed for controlling the position of an instantaneous portion of a cathode-ray-tube trace. Application of sampled-data theory in a radar display system.

[29] C. H. Knapp, E. Shapiro, and R. A. Thorpe, "The *Z*-Transform: A Method for Analysis of Sampled-Data Systems," IBM Res. Rep. RC-17; December, 1957.

Report available upon request to the editor, *IBM J. Res. and Dev.*, 590 Madison Ave., New York 22, N. Y.

[30] G. M. Kranc, "Input-output analysis of multirate feedback systems," IRE TRANS. ON AUTOMATIC CONTROL, no. PGAC-3, pp. 21–28; November, 1957.

Extension of *Z*-transform methods to sampled-data systems containing synchronized switches which operate at different sampling rates.

[31] G. M. Kranc, "Additional techniques for sampled-data feedback problems," 1957 WESCON CONVENTION RECORD, pt. 4, pp. 157–165.

Standard *Z*-transform methods are applied to the problem arising when sampling switches close for a duration time which is too long for the process to be described with sufficient accuracy by impulse modulation. The second problem deals with analysis of multirate systems, *i.e.*, when the sampling switches operate at different sampling rates.

[32] J. Kukel, "Sampling in linear and nonlinear feedback control systems," 1957 IRE NATIONAL CONVENTION RECORD, pt. 4, pp. 43–56.

Sampling in control systems is examined with the aid of the *Z* transformation. Covers systems with slow and fast sampling, with or without clamping. Linear, time varying parameter, and nonlinear systems in transient state are solved. Well-defined mathematical analysis throughout.

[33] G. V. Lago and J. G. Truxal, "The design of sampled-data feedback systems," *AIEE Trans.*, vol. 73, pt. 2, pp. 247–253; November, 1954.

Impulse response method offered as complementary to *Z*-transform synthesis method for simple first and second-order systems. Allows for critical study of the system response between sampling instants.

[34] G. V. Lago, "Additions to sampled-data theory," *Proc. NEC*, vol. 10, pp. 758–766; 1954.

Comparison of *Z*-transform (time response) and Laplace transform (frequency response) methods for design and synthesis of sampled-data systems. Author concludes *Z*-transform method is superior. Frequency response equations give equivalent results only under special conditions and suitable modifications. Basic agreement with conclusions reached by E. I. Jury.

[35] G. V. Lago, "Additions to *Z*-transformation theory for sampled-data systems," *AIEE Trans.*, vol. 73, pt. 2, pp. 403–408; January, 1955.

Review of *Z*-transform theory. Method outlined whereby output at any point in the sampling period can be found by modified *Z* transforms. Equations developed for error coefficients in terms of *Z* transforms.

D. Leiby. See H. A. Chestnut.

[36] A. B. Lees, "Interpolation and extrapolation of sampled-data," IRE TRANS. ON INFORMATION THEORY, vol. IT-2, pp. 12–17; March, 1956.

Extension of the optimum filtering theory of Zadeh and Ragazzini to the case where the time function to be operated upon is available only at a sequence of sampling instants.

[37] C. E. Lenz, "Analysis of a sampled-data servomechanism containing an asymmetric quadratic element," Ph.D. dissertation, Cornell Univ., Ithaca, N. Y., 150 pp.; 1957. Abstract published in *Dissertation Abstracts*, pp. 2958–2959; December, 1957.

Sampled-data systems treated wherein the feedback loop is closed for only a short time periodically, the resultant gain-control voltage being maintained until the next sample is taken. Reviews mathematical methods for analysis of sampled-data systems. Construction of an analog sampler with two synchronously operating sampling switches described.

D. P. Lindorff. See G. W. Johnson.

[38] W. K. Linvill, "Sampled-data control systems studied through comparison of sampling with amplitude modulation," *AIEE Trans.*, vol. 70, pt. 2, pp. 1779–1788; 1951.

Frequency-domain analysis. Treating the process of sampling as impulse modulation allows sampled-data system analysis by conventional frequency response techniques. All signals are described by continuous time functions and their transforms as contrasted to the *Z*-transform method where all signals are described by sequences of samples.

[39] M. Mori, "Root-locus method of pulse transfer function for sampled-data control systems," IRE TRANS. ON AUTOMATIC CONTROL, no. PGAC-3, pp. 13–20; November, 1957.

Examines the relationships between the root loci in the *S* and *Z* planes for continuous and sampled-data systems.

[40] M. Mori, "Statistical treatment of sampled-data control sys-

tems for actual random inputs," *ASME Trans.*, vol. 80, pp. 444–456; February, 1958.
Statistical relations indicated in terms of the correlation functions of time series, the pulse spectral densities, and modified Z transforms. Comparison is made between sampled-data and continuous-data systems.

[41] G. J. Murphy and R. D. Ormsby, "A survey of techniques for the analysis of sampled-data control systems," IRE Trans. on Automatic Control, no. PGAC-2, pp. 79–90; 1957.
Good bibliography included. (This reference is now out of print and can no longer be procured from the IRE.) Impulse response, frequency response, Z-transform techniques outlined.

C. G. Nordling. See G. W. Johnson.

[42] R. Oldenburger, ed., "Frequency Response," The Macmillan Co., New York, N. Y.; 1956.
Book surveys frequency response methods. Pt. 8, pp. 309–325, covers sampling controls. Contains English translation of Russian paper by Tsypkin on "Frequency method of analyzing intermittent regulating systems," and Barker [2].

R. D. Ormsby. See G. J. Murphy.

[43] J. R. Ragazzini and L. A. Zadeh, "The analysis of sampled-data systems," *AIEE Trans.*, vol. 71, pt. 2, pp. 225–234; November, 1952.
Unifies earlier Z-transform and S-transform approaches to sampled-data systems. Formulates input-output relations for various types of such systems and treats such systems by considering the sampler as a time-variant element. Concise mathematical summary of the analysis of sampled-data systems.

[44] J. R. Ragazzini, "Digital computers in feedback systems," 1957 IRE National Convention Record, pt. 4, pp. 33–42; 1957.
Emphasis on the design of digital controllers for linear sampled-data systems.

J. R. Ragazzini. See A. R. Bergen.

J. R. Ragazzini. See J. Sklansky.

[45] J. M. Salzer, "Signal flow reductions in sampled-data systems," 1957 WESCON Convention Record, pt. 4, pp. 166–170.
Signal flow graphical method suggested as replacement for conventional block diagrams used in control system design. Author believes greater clarity and ease of operation justifies the signal flow technique.

W. Schroeder. See E. I. Jury.

E. Shapiro. See C. H. Knapp.

[46] J. Sklansky and J. R. Ragazzini, "Analysis of errors in sampled-data feedback systems," *AIEE Trans.*, vol. 74, pt. 2, pp. 65–71; May, 1955.
Sampled-data system errors result from imperfect response of the system to an applied input. One possibility occurs from lags or leads of the continuous part of the system. Another error source is the sampler which causes "ripple." This "ripple" contains those frequency components which are at the sampling frequency or its harmonics and causes degradation in performance. Formulation of system error obtained by application of both Laplace and Z transforms.

[47] J. Sklansky, "Network compensation of error-sampled feedback systems," Ph.D. dissertation, Dept. of Elec. Eng., Columbia Univ., New York, N. Y.; 1955. Also published as Tech. Rep. T-7/B by Columbia Univ. Electronic Res. Labs.

[48] B. H. Stafford, "Frequency Analysis of Some Closed-Cycle Sampled-Data Control Systems," Naval Res. Lab. Rep. 3910; January, 1952. (Available from ASTIA, the Armed Services Technical Information Agency, to qualified government contractors.) Not reviewed.

[49] A. K. Susskind, ed., "Notes on Analog-Digital Conversion Techniques," M.I.T. Servomechanisms Lab., Cambridge, Mass.; 1958.
New book prepared by staff members of the M.I.T. Servomechanisms Laboratory. Chapter II: sampling and quantizing. Covers basic concepts of linear system analysis, sampling as impulse modulation, recovery of a signal from its samples. Brief treatment of sampled-data system analysis techniques.

R. A. Thorpe. See C. H. Knapp.

[50] B. M. Tostanoski, "The analysis of sampled-data servomechanisms performed on the IBM type 650," *Commun. and Electronics*, no. 26, pp. 446–450; September, 1956.
Exact response of linear systems which combine continuous and discrete filters can be obtained for discrete values of time by series of well-defined mathematical processes; use of digital computer in analysis of these systems.

[51] J. Tou, "Digital compensation for control and simulation," Proc. IRE, vol. 45, pp. 1243–1248; September, 1957.
Digital programming compensation. Z-transform method reviewed. System stability improved by making use of the computer to perform information programming or data processing.

[52] N. W. Trembath, "Random-Signal Analysis of Linear and Nonlinear Sampled-Data Systems," M.I.T. Dynamic Analysis and Control Lab. Rep. 108, Contract AF33(616)3398. AD-141051. (Available from ASTIA, the Armed Services Technical Information Agency, to qualified government contractors.)
Method for computing the response at all times of a sampled-data system to an excitation of arbitrary waveform. Convolution equations are derived which express time-domain relationships between system input and output signals.

[53] J. G. Truxal, "Automatic Feedback Control System Synthesis," McGraw-Hill Book Co., Inc., New York, N. Y.; 1955.
Chapter 9, pp. 500–558: sampled-data control systems. Covers analysis of the sampler, smoothing, Z-transform theory, combinations of networks, analytical determination of stability, graphical stability analysis, continuous and sampled-data systems with transportation lags.

J. G. Truxal. See G. V. Lago.

L. A. Zadeh. See J. R. Ragazzini.

Contributors

This is a new section which introduces the authors to the PGAC readers. It is planned to continue this feature in each issue.—The Editor

John A. Aseltine was born on April 12, 1925, in Palo Alto, Calif. He received the B.A. degree in physics in 1947 from the University of California, the M.S. degree in engineering in 1949, and the Ph.D. degree in engineering in 1952 from U.C.L.A.

J. A. Aseltine

In 1952 Dr. Aseltine joined the Hughes Aircraft Company, to work on missile systems analysis. In 1955 he helped found Aeronutronic Systems, Inc., Los Angeles, Calif. He is presently on the technical staff of Space Technology Laboratories, Los Angeles.

Since 1951 he has been a lecturer in engineering at UCLA, where he teaches in the field of control systems. His book "Transform Method in Linear System Analysis" was published by McGraw-Hill Book Company, Inc., in 1958.

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Ze'ev Bonenn (A'51–M'56) was born in Jerusalem, Israel, on October 29, 1926. He received the degree of electrical engineer from the Technion, Israel Institute of Technology, Haifa, Israel, in 1951.

During 1951–1952 he worked in the United States under the foreign exchange program of the General Electric Company.

Z. Bonenn

Since 1952 he has been with the Ministry of Defense, Israel, working on nonlinear control systems. In 1957–1958 he lectured on automatic control in the Technion.

Mr. Bonenn is a member of the Association of Engineers and Architects in Israel.

A. Robert Mancini (M'55) was born in Detroit, Mich., on March 1, 1924. He received the B.S. degree in 1949 and the M.S. degree in 1957, both in engineering, from U.C.L.A. His thesis concerned the feasibility of conditional feedback flight control systems.

A. R. Mancini

As a member of the Electrical Group, Engineering Research Laboratory, North American Aviation, Inc., he worked on instrumentation for flight control system laboratory tests. From 1951 to 1955 he was with the Bendix Aviation Corporation, packaging telemetering equipment for missile systems; he also designed, built, and tested servomechanisms for sonar and shipboard fire control systems. Since 1955 Mr. Mancini has been working in the missile and aircraft flight control field. During this period he has been associated with the Missile Systems Division of the Lockheed Aircraft Corporation, Radioplane Company, and Aeronutronic Systems, Inc. During the past two years he has been engaged in the development of self-optimizing control systems and missile guidance and control systems.

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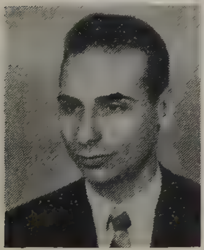
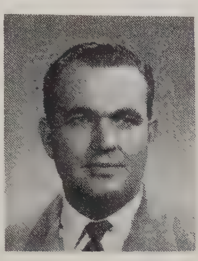
Arthur Mayer was born on November 24, 1925, in New York, N. Y. He received the B.S. degree in physics from the College of the City of New York in 1947 and the M.A. degree in physics from Harvard University, Cambridge, Mass., in 1948. He served for a year as a research assistant at the Los Alamos Scientific Laboratory. Since 1949 he has been employed at the Reeves Instrument Corporation, Garden City, N. Y., where he is now a senior staff member.



A. Mayer

Mr. Mayer has been engaged in the design and analysis of tracking and guidance systems. He has also contributed to the development of unusual analog computers, and has several patents in the field of electromechanical components.

Michael G. Rekoff, Jr., (A'56) was born on July 27, 1929, in Galveston, Tex. He received the B.S. degree in electrical engineering in June, 1951, from Texas A. & M., College Station, Tex. From 1951 to 1953 he served in the U. S. Army Signal Corps. In June, 1955, he received the M.S. in electrical engineering from Texas A. & M., where he was an instructor as well as a graduate student. Since then, he has been an instructor and graduate student at the University of Wisconsin.



M. G. Rekoff, Jr.

Mr. Rekoff is a registered professional engineer in Wisconsin and a member of the AIEE and Eta Kappa Nu.

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Charles W. Sarture was born in Gary, Ind., on March 5, 1928. He received the B.S., M.S., and Ph.D. degrees in electrical engineering from Purdue University, Lafayette, Ind., in 1950, 1955, and 1956, respectively. At Purdue, where he was both a graduate assistant and an IBM Fellow, his doctoral research was directed toward the determination of power requirements for feedback controls.

C. W. Sarture

From 1950 to 1952 Dr. Sarture served in the U. S. Army and was attached to the Martin Company in connection with the Missilemaster Program. From 1952 to 1953 he was associated with Armour Research Foundation, Chicago, Ill., where he was engaged in servomechanism analysis.

Dr. Sarture was with Aeronutronic Systems, Inc., Glendale, Calif., from 1956 to 1958. There he was engaged in research on adaptive autopilots. In July, 1958 he joined Space Technology Laboratories, Los Angeles, Calif., where he is now head of the analysis and simulation section. He is a lecturer in engineering at the University of California at Los Angeles.

Dr. Sarture is a member of Tau Beta Pi, Sigma Xi, Eta Kappa Nu, and the AIEE.

Peter R. Stromer was born in Norwich, Conn., on July 16, 1929. He received the B.A. degree in library science from Syracuse University, Syracuse, N. Y., in 1952.

P. R. Stromer

Since 1952 his professional library experience has been with libraries covering various physical sciences. He served as reference and circulation librarian at the Aerodynamics Laboratory, David Taylor Model Basin, U. S. Navy Department, Washington, D. C., for two years, and transferred to the position of librarian, Research and Development Department, U. S. Naval Powder Factory, Indian Head, Md., in 1954. Since 1956 he has been librarian in charge of the Light Military Electronics Department Library, General Electric Company, Schenectady, N. Y.

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Charles F. White (S'35–A'37–M'47) was born in Columbus, N. M., on August 4, 1913. He received the B.S. degree in engineering in 1935, and the M.S. degree in electrical engineering in 1938, both from the University of California, Berkeley, Calif.

C. F. White

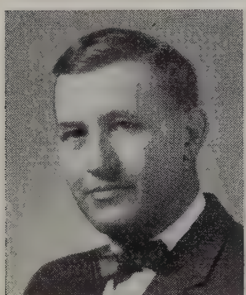
After teaching as an instructor in electrical engineering at the University of California and Yale University, Mr. White was employed by the Navy Department, Bureau of Ordnance, and later by the U. S. Signal Corps. Following his completion of the Navy's radar courses at Bowdoin College, Massachusetts Institute of Technology, and Bell Laboratory School for War Training, he served as an officer at the Naval Research Laboratory in Fire Control Radar system development (January, 1944 to June, 1946).

After his return to civilian life, Mr. White remained at the Naval Research Laboratory in the Equipment Research Branch, Radar Division, where he has specialized in feedback systems and networks.

The PGAC Administrative Committee July, 1958–July, 1959

To acquaint the members of the PGAC with their Administrative Committee the following biographies have been compiled. It is planned to make this an annual feature of our TRANSACTIONS. We are indebted to Harold Levenstein, a member of the committee, for procuring and arranging the material.—*The Editor.*

John E. Ward (A'52–M'57) was born on January 4, 1920, in Toledo, Ohio. He attended the University of Toledo from 1938 to 1940, and received the B.S. and M.S. degrees in electrical engineering from the Massachusetts Institute of Technology, Cambridge, Mass., in 1943 and 1947, respectively. From 1943 to 1945 he was a staff member at the M.I.T. Radiation Laboratory, where his work included radar homing systems for glide bombs, design of radar test equipment, and a radar bomb scoring system.



J. E. WARD
Chairman
1958–1959

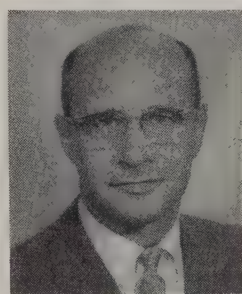
In 1945 he joined the M.I.T. Servomechanisms Laboratory, where he received successive appointments as research assistant, staff member, and project engineer. In 1955 he received his present appointment as Executive Officer. His work has included servo design, systems design for bomber turret, analog and digital flight test instrumentation for airborne accuracy tests of fire control systems, design of servo data repeaters, analog-to-digital conversion devices, integration of digital computers in control and data processing systems.

He is co-author of "Notes on Analog-Digital Conversion Techniques," published by John Wiley and Sons, New York, N. Y., in 1957, and has written and contributed to several papers which have appeared in IRE, ASME, and AIEE publications, and in *Control Engineering.*

He has served on the IRE Standards Committee and as chairman of Technical Committee 26 on Feedback Control Systems. He is a member of Sigma Xi.

John M. Salzer (S'48–A'50–M'57) was born on September 12, 1917, in Vienna, Austria. He received the B.S. and M.S. degrees in electrical engineering from the Case Institute of Technology, Cleveland, Ohio, in 1947 and 1948, respectively, and the Sc.D. degree from the Massachusetts Institute of Technology, Cambridge, Mass., in 1951.

In 1948 he was appointed research associate at the M.I.T. Servomechanism Laboratory where he worked on the design and application of the Whirlwind computer. He joined the Hughes Aircraft Company, Culver City, Calif., in 1951 as a group head working on interception system analysis, incorporation of digital computers into airborne systems, and programming and system coordination. In 1954 he was employed by the Magnavox Company's Research Laboratories, Los Angeles, Calif., as assistant director of research, where he took a key role in building the laboratory from a twenty-man research group to one of more than one hundred men within a year. Currently director of systems with Magnavox, he is now in charge of system investigations and promotion of military products. His work here includes supervision of missile guidance projects, control computers, and communication systems.



J. M. SALZER
Vice-Chairman
1958–1959

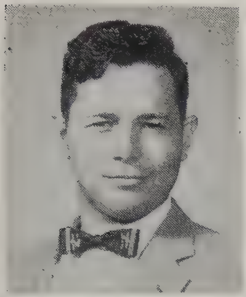
Dr. Salzer is the author and co-author of numerous papers and articles concerning analog and digital computers and controls, several of which appeared in *Control Engineering* and the PROCEEDINGS OF THE IRE.

Dr. Salzer has previously served as vice-chairman of the Los Angeles PGEC; in PGAC he has been vice-chairman and chairman of the Los Angeles Section, and national secretary. He is a member of Tau Beta Pi, Eta Kappa Nu, Sigma Xi, AIEE, ACM, and AOA.

George A. Biernson (A'53) was born on April 11, 1927, in Massachusetts. He received the B.S. degree in electrical engineering in 1948 and the M.S. degree in economics in 1949, both from the Massachusetts Institute of Technology, Cambridge, Mass.

After a year as an instructor at the University of

Maine in Orono, he became research engineer at the M.I.T. Servomechanisms Laboratory in June, 1950. There he worked on performing research and development in feedback control systems.

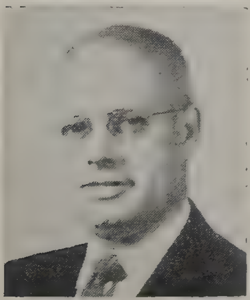


G. A. Biernson
Secretary-Treasurer
1958–1959

In April, 1956, he became associated with the Sylvania Electric Company, in Waltham, Mass., where he is presently an engineering specialist working on radar and missile systems.

Mr. Biernson is the author of many papers on feedback control in AIEE and IRE periodicals. He is a former chapter chairman of the Boston Section of the PGAC.

George S. Axelby (S'50–A'51–M'54–SM'57) was born in Thomaston, Conn., on March 7, 1922. He received the B.S. degree in electrical engineering from the University of Connecticut, Storrs, Conn., in 1950, and the M.E. degree in electrical engineering from Yale University, New Haven, Conn., in 1951 on a Westinghouse Fellowship. From 1939 to 1949 Mr. Axelby worked for the Seth Thomas Clock Company and Stromberg Time Corporation as a draftsman and designer of electromechanical timing devices and remote control systems.



G. S. Axelby

Since 1951 he has been engaged in the design of feedback control systems at Westinghouse in the Air Arm Division, Baltimore, Md., where he currently has the position of Fellow Engineer.

Mr. Axelby is the author of several papers pertaining to feedback control systems. At present he is editor of the PGAC Transactions, a member of IRE Committee 26.0 on Feedback Control, chairman of Subcommittee 26.1, and technical program chairman of the 1960 Automatic Control Conference in Dallas, Tex. He was the founder of the Baltimore chapter of the PGAC, and has served as the chapter chairman; at the present time he is on the chapter's steering committee.

Mr. Axelby is a member of the AIEE, Eta Kappa Nu, and Tau Beta Pi.

Victor Azgapetian (SM'55) was born on May 26, 1919, in New York, N. Y. He received the B.S. degree in 1947 and the M.S. degree in 1948, both in electrical engineering, from the Massachusetts Institute of Technology, Cambridge, Mass. He served for a period of years as chief (and sole) engineer with the M.I.T. Cyclotron Group.

V. Azgapetian

After an employment of two years with the Sperry Gyroscope Corporation, Garden City, N. Y., he joined Servomechanisms Inc., and now holds the position of chief systems engineer at its Research Laboratory in Goleta, Calif.

Mr. Azgapetian's work has been chiefly in the field of computers and computer design.

Nasli H. Choksy (A'50–M'55) was born in Bombay, India, on May 12, 1926. After completing his elementary education there he arrived in the United States in 1947 and obtained the B.S. degree from the Milwaukee School of Engineering, Milwaukee, Wis., in 1950. He received the M.S. and Ph.D. degrees from the University of Wisconsin, Madison, Wis. in 1952 and 1955, respectively. All his degrees were obtained in the field of electrical engineering.

N. H. Choksy

Apart from part-time engineering employment and summer employment, Dr. Choksy's first position was the one he has held since 1955—assistant professor of electrical engineering at The Johns Hopkins University, Baltimore, Md., where he teaches graduate courses including those in his main field of interest, servomechanisms theory.

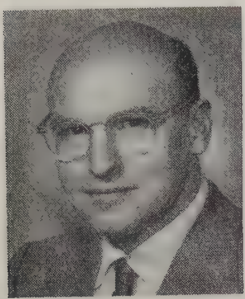
He has published papers on cable fault detection and on cybernetics in journals in India. His doctoral dissertation was on the stability of time-lag systems.

He has served as chairman of the Baltimore Chapter of the PGAC.

Dr. Choksy is an associate member of the AIEE, and is a member of Sigma Xi, the Mathematical Association of America, the American Association of University Professors, the Society for Industrial and Applied Mathematics, the Tensor Club of Great Britain, the Research Association of Applied Geometry (Japan), and the Society for General Systems Research.

Eugene M. Grabbe (A'54) was born on December 4, 1912, in Johnstown, Pa. He received the B.S. degree in mathematics from Duke University, Durham, N. C., in

1935, the M.S. in physics from Brown University, Providence, R. I., in 1937, and the Ph.D. degree in physics from Yale University, New Haven, Conn., in 1949, for his dissertation on ferromagnetism.



E. M. Grabbe

He was employed by the U. S. Rubber Company as a research physicist at their general laboratories, Passaic, N. J., from 1939 to 1945, where his work concerned physical properties and testing of high polymers (rubber, plastics, and fibers). During the following three years he was technical consultant on new product development with the Homelite Corporation, Portchester, N. Y. In 1945 he joined Hughes Aircraft Company, Culver City, Calif., as associate head of the Computer Systems Department in the Research and Development Laboratories.

Since 1954 he has been Senior Staff Consultant on Automation at The Ramo-Wooldridge Corporation, Los Angeles, Calif.

He was the project head for the first airborne digital computer for automatic flight control, and is also experienced in business data processing and automatic control systems. His publications include various technical papers on digital computers, electronic data processing, automation and related fields. He was editor and contributor for "Automation in Business and Industry," published by John Wiley and Sons, Inc., New York, N. Y., in 1957, and was co-editor, with S. Ramo and D. E. Wooldridge, of "Handbook of Automation and Control," also published by Wiley. He was a member of a team of United States technical personnel who visited Russia in August. A consulting editor to *Control Engineering*, he also lectures at the University of California at Los Angeles.

Dr. Grabbe organized the Los Angeles Chapter of PGAC, now one of the largest in the country, and was chairman of the PGAC Administrative Committee. He is a member of Phi Beta Kappa and Sigma Xi.

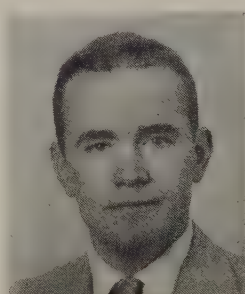
Harold Levenstein (A'46–M'55) was born on June 28, 1923, in Philadelphia, Pa. He received the B.E.E. degree from Cooper Union, New York, N. Y., in 1943 and the M.E.E. degree from the Polytechnic Institute of Brooklyn, N. Y., in 1949. In addition to course work for the doctorate, he has been a special student in fire control systems and communications theory at Massachusetts Institute of Technology, Cambridge, Mass., in 1949, 1953, and 1955.

H. Levenstein

He joined the Western Electric Company, Bayonne, N. J., in 1943, as a test planning engineer on radar systems. In 1946 he became a member of the Sight Laboratory, Fairchild Camera and Instrument Company, working in fire control systems. From 1947 to 1950 he was employed by Fairchild Guided Missiles Division as an electronics engineer on telemetering and guidance systems for the Lark missile. In 1950 he joined the W. L. Maxson Corporation, New York, N. Y., as a senior engineer, responsible for fire control and bombing and navigation systems. He has been manager of the Analysis Group and the Systems Engineering Department. In 1955 he took a leave of absence to work as a member of the M.I.T. Instrumentation Laboratory on inertial navigation systems. In June, 1956, he rejoined Maxson as research assistant to the vice-president of the Research and Development Division. He acted as consultant in systems and control engineering and was responsible for preliminary system planning. Currently Mr. Levenstein is director of the Research Laboratory, generally responsible for systems analysis and research functions of the Research and Development Division.

He is a member of Tau Beta Pi, Sigma Xi, the American Physical Society, and the American Mathematical Society. He is chairman of the Long Island Chapter of the PGAC.

David P. Lindorff (A'51–M'55) was born September 9, 1922, in Flushing, N. Y. He received the B.S. degree in electrical engineering from the Massachusetts Institute of Technology, Cambridge, Mass., in 1948, and the M.S. degree from the University of Pennsylvania in 1950.



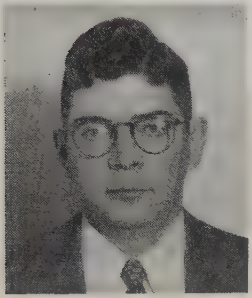
D. P. Lindorff

During World War II he was a staff member at the M.I.T. Radiation Laboratory. He was employed as project engineer at the Naval Communications Laboratory in Washington, D. C., in 1949. In 1950 he held the position of instructor of electrical engineering at Purdue University, Lafayette, Ind., and subsequently joined the staff of the Electrical Engineering Research Laboratory, under sponsorship of the IBM Corporation.

Professor Lindorff's primary interests in teaching and research are in the field of automatic control, particularly in the area of sampled-data systems. He has served as consultant to a number of engineering firms, and as a member of an ad hoc advisory committee to the Associated Universities, Inc., in connection with the control aspects of the Greenbank Radio Telescope installation. Recently while on sabbatical leave he spent six months doing research at the University of Cambridge, Eng. He has published papers on analytical methods in sample-data control systems and instrumentation.

He is a member of the IRE Technical Committee 26 on Feedback Control Systems, and Sigma Xi, Eta Kappa Nu, and the AIEE.

John C. Lozier (SM'45) was born February 5, 1912, in Bayside, N. Y. He received the A.B. degree with a major in physics from Columbia University, New York, N. Y., in 1934, and continued graduate work in physics at Princeton University, Princeton, N. J., in 1935.

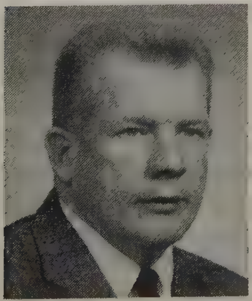


J. C. Lozier

After a year with RCA, he joined the Transmission Development Department of the Bell Telephone Laboratories, New York, N. Y., in 1936, where he worked on transmission theory, modulation theory, and nonlinear feedback systems. During the war he worked on homing missiles and developed a quasilinear approach to the analysis and design of on-off control systems. In 1953 he joined the Bell Telephone Laboratories Military Systems Studies Department, where he is responsible for research and consulting activities in digital computing and control systems.

He is vice-chairman of the American Automatic Control Council, to which he is IRE delegate. He is a former chairman of the PGAC, a member of IRE Technical Committee 26 on Feedback Control Systems, and a former chairman of one of its subcommittees.

Thomas F. Mahoney (S'43–A'46–M'50) was born in Norwood, Mass., on January 28, 1922. He received the B.S. degree in electrical engineering with high honors from Northeastern University, Boston, Mass., in 1944, after specializing in communications. He received further education in radar at the Harvard and M.I.T. Naval Training School, Cambridge, Mass., in 1944, and at the Bell Telephone Laboratories School, in 1945. As research assistant, he specialized in servomechanisms at the M.I.T. Servomechanisms Laboratory from 1946 to 1948. He is presently manager of the Servo-Mechanics Section in the Airborne Radar Development at the Raytheon Manufacturing Company, Waltham, Mass.



T. F. Mahoney

Mr. Mahoney has been active in both the IRE and the AIEE. He was elected to the PGAC Administrative Committee in 1955 and served as PGAC secretary from 1955 to 1956. He is now chairman of the Subcommittee for Types and Components Terminology for Automatic Control in the American Standards Association.

Harvey A. Miller (SM'58) was born in Rutland, N. D., on May 18, 1923. He received the B.S. degree in electrical engineering from the University of Minnesota, Minneapolis, in 1944. Following service in the U. S. Navy, he became a teaching assistant and then assistant scientist at the University of Minnesota while pursuing graduate studies. After receiving the M.S. degree in electrical engineering in 1948 he joined the faculty as instructor in electrical engineering.

H. A. Miller

He was in operating charge of the analog and digital computation center until 1952, when he joined the Research Division of Raytheon Manufacturing Company, Waltham, Mass. There his work concerned magnetic amplifiers, servomechanisms, and computers resulting in several technical papers. He became group manager of the Applied Electronics Group in the Research Division and worked on transistor circuits, computers, automatic controls, and ultrasonics. In 1958 he joined the Advanced Development Group of the Government Equipment Division of Raytheon, as Manager of Computer Development.

Mr. Miller is a member of Tau Beta Pi, Eta Kappa Nu, Kappa Eta Kappa, and the AIEE.

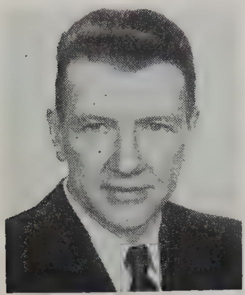
James H. Miller (S'47–A'49–M'54) was born on May 31, 1923, in Des Moines, Iowa. He received the B.S. degree in electrical engineering from Iowa State College, Ames, in 1948, and attended graduate school at The Johns Hopkins University, Baltimore, Md., and the University of Akron and Kent State University, Ohio.

In 1948 he joined the Glenn L. Martin Company, Baltimore, Md., as an electromechanical engineer and was in charge of servomechanisms development for a guided missile. Since 1952 he has been employed by the Goodyear Aircraft Corporation in Akron, as an engineer specialist working on missile guidance systems.

Mr. Miller organized the Akron Chapter of the PGAC.

O. Hugo Schuck (A'34–M'44–SM'49–F'57) was born September 20, 1909, in Philadelphia, Pa. He received the B.S. and M.S. degrees in electrical engineering from the University of Pennsylvania. After completing his graduate studies in 1932 he was an assistant in the aeronautics department of the Franklin Institute Museum in Philadelphia. He was awarded a professional electrical engineering degree from the University of Pennsylvania in 1938. In 1934 he joined the International Resistance Company, Philadelphia, as a development engineer, and from 1937 to 1941 he was employed as an acoustical engineer for C. G. Conn, Ltd., Elkhart, Ind.

During World War II he was associated with the Harvard University Underwater Sound Laboratory and the M.I.T. Radiation Laboratory in Cambridge, Mass. His duties included high priority scientific work with sonar devices for antisubmarine warfare and radar fire control devices designed to halt Japanese suicide plane attacks. His contributions in these fields were recognized by the Certificate of Appreciation award in 1947.



O. H. Schuck

He joined the Minneapolis-Honeywell Regulator Company in 1945 as a senior research engineer, and within several months was asked to manage a new emphasis on research in aeronautical control and guidance fields. In 1954 he was named director of research for the company's Aeronautical Division, formed in 1949. This department now includes more than one hundred research scientists and engineers, plus technicians and support personnel; it is the largest of Honeywell's divisional research staffs. Much of the department's effort is directed at flight control, navigation, and instrumentation problems, which may become production potential for the Aeronautical Division.

More than thirty patents have been registered for Mr. Schuck as sole or joint inventor. Most are in the aeronautical, acoustical, underwater sound, and instrumentation fields. He has authored eight published papers.

He is a Fellow of the Acoustical Society of America, a member of the AIEE and the AIEE Committee on Feedback Control systems, and a member of the Subcommittee on Stability and Control of the National Advisory Committee for Aeronautics. Recently he resigned as a member of the Assistant Secretary of Defense's advisory group on electronic parts. He is also a member of Sigma Xi, Tau Beta Pi, Eta Kappa Nu, Pi Mu Epsilon, and Sigma Tau.

Otto J. M. Smith (M'44–SM'51) was born on August 6, 1917, in Urbana, Ill. He received the B.S. degree in chemistry from Oklahoma Agricultural and Mechanical College, Stillwater, Okla., and the B.S. degree in electrical engineering from the University of Oklahoma, Norman, Okla., in 1938. He was a research assistant at the H. G. Ryan High Voltage Laboratory at Stanford University, Stanford, Calif., from 1938 until 1941, when he received the Ph.D. degree in electrical engineering.

O. J. M. Smith

He has taught and conducted research at Tufts College, Doble Engineering Company, Denver University, Westinghouse Research Laboratory, Summit Research and Development Laboratory and Shell Development Company. He has been at the University of California, Berkeley, since 1947. From 1954 to 1956, while on leave from the University of California, he was a visiting professor of servomechanisms at the Instituto Tecnologico de Aeronautica, Sao Jose dos Campos, Estado de Sao Paulo, Brazil. He also studied the educational system of Brazil, made recommendations, and taught advanced automatic control systems for the Brazilian government. He is presently professor of electrical engineering at the University of California, Berkeley.

He is the author of "Feedback Control Systems," and the inventor of a low-frequency sine-function generator, an X-ray thickness gauge, dead-beat predictor controls for automatic systems, a constant-frequency variable-speed generator, a controlled-torque motor, and a phase-shift scaler. He has published research in the fields of magnetic amplifiers, magnetic frequency multipliers, semiconductors, phonograph recording, high-voltage corona, power-line fault locators, radiation instrumentation, counters, education, economic analogs, nonlinear feedback systems, and statistics.

Dr. Smith, a Fellow of the American Association for the Advancement of Science, is active on editorial and review committees of AIEE, PGAC, and IRE. He is a member of the American Society for Engineering Education, American Physical Society, American Institute of Physics, Sigma Xi, Phi Kappa Phi, Tau Beta Pi, Eta Kappa Nu, Phi Lambda Upsilon, Alpha Phi Omega, Kappa Tau Pi, Phi Eta Sigma, and Alpha Tau Omega.

Thomas M. Stout (A'49–SM'54) was born on November 26, 1925, in Ann Arbor, Mich. He received the B.S. degree in electrical engineering from Iowa State College, Ames, in 1946, and the M.S. and Ph.D. degrees from the University of Michigan, Ann Arbor, in 1947 and 1954, respectively.

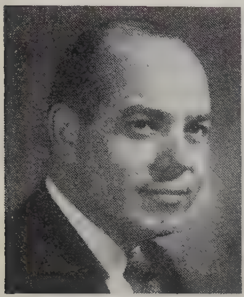
T. M. Stout

From 1947 to 1948 he was employed by Emerson Electric Manufacturing Company, St. Louis, Mo. In 1948 he joined the University of Washington, Seattle, as an instructor and later as an assistant professor of electrical engineering, teaching graduate courses in servomechanisms and computers.

In 1954 he joined the Schlumberger Instrument Company, Ridgefield, Conn., where his work concerned the study of nonlinear control systems and measurement problems. He has been employed by the Thompson-Ramo-Wooldridge Products Company, Los Angeles, Calif., since 1956, studying the design of computer control systems for industrial processes. He is the author of a number of papers concerning analysis of nonlinear feedback systems, optimum relay servomechanisms, and process control systems employing digital computers.

Dr. Stout is a member of the AIEE, the American Association for the Advancement of Science, the American Society for Engineering Education, and the ISA.

Arthur R. Teasdale (A'50–SM'54) was born on September 3, 1919, in Austin, Tex. He received the B.S. and M.S. degrees in electrical engineering from the University of Texas, Austin, in 1942 and 1950. From 1942 until 1946 he worked with General Electric's Advanced Engineering Program in Schenectady, N. Y., first on rotating test assignments, and then in a teaching capacity. In 1947 he returned to the University of Texas as an assistant professor in electrical engineering; two years later he became an associate professor.



A. R. Teasdale

From 1951 until 1954 he was employed by the Consolidated Vultee Aircraft Corporation, Fort Worth, Tex., as a design specialist in feedback controls and group engineer in technical design. While with Convair his responsibilities included the design, construction, and testing of nuclear reactor control systems and a nuclear reactor simulator, and technical design of navigation, bombing, missile guidance, and nuclear control systems.

Since 1952 he has been a member of the graduate faculty of electrical engineering at Southern Methodist University, Dallas, Tex. In 1954 he joined the Temco Aircraft Corporation in Dallas as chief electronics designer, and in 1955 he was put in charge of Systems Research. In 1957 he became Chief of the Avionics Department in charge of Temco's Electronic Development; as director of advanced technology, he is presently concerned with advanced studies in systems analysis.

Mr. Teasdale is Secretary for the Southwest IRE Conference, 1959. Also active in the AIEE Committee on Feedback Control, and a registered professional engineer in the state of Texas, he is a member of Tau Beta Pi, Eta Kappa Nu, and Rho Epsilon Delta. He is coauthor and contributor to numerous technical papers for the Defense Department and professional magazines.

Robert B. Wilcox (M'50) was born on March 20, 1921, in Dartmouth, Nova Scotia, Can. He received the B.E.E. degree from Nova Scotia Technical College in 1944. He then served with the Royal Canadian Navy as a senior electrical officer on active service with the North Atlantic Striking Group until 1946. He resumed his technical training in 1946 at the Massachusetts Institute of Technology, Cambridge, Mass., receiving the M.S. degree in 1948. While at M.I.T. he was a member of the faculty, teaching graduate servomechanisms courses. He joined the Raytheon Manufacturing Company, Waltham, Mass., in 1948, as manager of the servomechanisms department and then as staff engineer. His work at Raytheon concerned servomechanisms for radar, countermeasures, missiles, and industrial control systems.

R. B. Wilcox

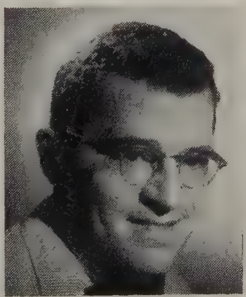
From 1956 until 1958 he was the director of the Control Systems Groups, Hycon Eastern, Inc., Cambridge, Mass., and directed an ICBM re-entry guidance project. He is presently with Sylvania Electric Products, Inc., Waltham, as manager of the projects department of the Missile Systems Laboratory. His work concerns antimissiles, radar, and AICBM development.

He has been a lecturer at Northeastern University, Boston, Mass., since 1952, teaching graduate courses on servomechanism theory and advanced feedback control systems. He is the author of "Design Techniques," which appears in the "Engineer's Handbook," edited by John G. Truxal. He has written and contributed to several publications on servomechanisms.

Mr. Wilcox assisted in organizing the PGAC, and served as chairman from 1954 to 1956. He is chairman of IRE Technical Committee 26 on Feedback Control Systems, and a member of the IRE Standards Committee, and the American Standards Association. He is the recipient of the Box Walter award from Dalhousie University, Halifax, N. S., the Engineering Institute of Canada Award in 1943, and the Nova Scotia Technical College Alumni Medal in 1944.

He belongs to Phi Delta Theta and Sigma Xi.

Felix Zweig (A'52) was born in Fort Wayne, Ind., on September 25, 1916. He received the B.E. degree in electrical engineering in 1938 and the Ph.D. degree in 1941, both from Yale University, New Haven, Conn. Except for a period at M.I.T. as a research associate during 1946, he has been engaged in teaching and research at Yale ever since. He was named professor of electrical engineering in 1955. He has been a consultant to the Whitney Blake Company, Burroughs Company, General Electric Company, and General Precision Laboratory. He has written on throttling hydraulic systems, Doppler navigation, and inertial navigation.



F. Zweig

Dr. Zweig has been a member of the AIEE Committee on Feedback Control Systems and the IRE Feedback Control Systems Committee. He is a Fellow of Timothy Dwight College at Yale, a member of the ASEE, Tau Beta Pi, and a past president of the Yale Chapter of Sigma Xi.

Correspondence

Standard Terminology*

An article in the March issue of these TRANSACTIONS requested opinions on proposed Standard terminology for feedback control systems.[1]

In the definition of Difference Transfer Ratio, I suggest that the definition be generalized by replacing the words "difference signal" by "difference between the input and output quantities." This particular term affects the definitions of Acceleration Constant, Acceleration Error Coefficient, Error Coefficient, Position Constant, Position Error Coefficient, Velocity Constant, and Velocity Error Coefficient. In some systems, the difference signal does not represent the difference between input and output quantities. As examples, King[2] describes systems where the negative feedback is not unity and Bailey[3] mentions positioning servos for gun drives in which the feedback transducer reflects motor position rather than output position. It is true that there is difficulty in verifying results by trying to measure the difference between input and output quantities in those systems where this difference does not appear as a signal. However it is felt that the more generalized definition better satisfies the usual error coefficient concept of predicting over-all system performance rather than performance at an internal error measuring point.

I would like to suggest also that consideration be given to the addition of wording in the definitions for Acceleration Error Coefficient, Position Error Coefficient, and Velocity Error Coefficient to indicate that these particular definitions are intended only for use with positioning servos, that is with those control systems where the differences are expressed in linear or rotational units of position rather than of velocity, etc.

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* Received by the PGAC, April 22, 1958.

[1] "Proposed IRE Standard terminology for feedback control systems," IRE TRANS. ON AUTOMATIC CONTROL, no. PGAC-4, p. 31; March, 1958.

[2] L. H. King, "Reduction of forced error in closed-loop systems," PROC. IRE, vol. 41, pp. 1037–1042; August, 1953.

[3] F. M. Bailey, "Performance of drive members in feedback control systems," IRE TRANS. ON AUTOMATIC CONTROL, no. PGAC-1, pp. 74–83; May, 1956. See p. 76.

Operations Research for Computer Control*

Before a computer can be used to control an industrial machine or process, the mode of operation of the machine or process must first be expressed as a mathematical equation. Control engineers are familiar with computers and servomechanisms, but are not especially able at writing the process descriptive equations on which computer control is based. Writing the mathematical models of a machine or process, and thereby making the machine or process amenable to automatic computer control, is a job for Operations Research.

The concept of basing control of an industrial process or machine (hereafter *process* includes machines too) on the automatic solution of process descriptive equations has existed for some time now. Obviously, before process equations can be solved, they must first be determined. This is the soft spot in computer control. Who can set up the necessary equations?

The engineering arts of putting equipment together, to solve almost any equation, are already highly developed. But the control engineer does not necessarily have the skills of analyzing a process and reducing it into the form of a mathematical model. This is why computer control, a powerful technique long recognized, has developed so slowly.

On the other hand, some OR aspirants claim that there are insufficient demands for their OR skills. So we have the situation that computer control suffers from lack of mathematically oriented process analysts, while some would-be OR workers can't find enough to do along such lines. What the process computer-control designer needs is the aid of the professional equation writer. And who can this man be if not the OR mathematician?

We don't mean to say that the engineer is the potential client of the mathematician. The way we see it, the engineers and applied mathematicians should both group under the banner of OR, and make a combined sally against the processing and fabrication industries. The big gun here is computer control.

Briefly, here is the line of reasoning we followed that leads to our strong feeling for computer control. We observed that all work consists of but two basic elements: the *energy* that does the actual work, and the *information* on how this work is done. For all practical purposes *energy* is about fully mechanized, for there is little use for muscle power nowadays. But we are just now entering into the era of mechanization of information, which is accomplished by automatic control systems, including computer elements.

"Routine means machine." That is, machines do routine work well, while men do it badly. So all routine work, physical and mental, is ripe for mechanization. Actively searching out routine industrial mental operations does lead to their mechanization. Examples are the mechanization of the mental functions of discrimination, classification, logic, and algebraic computation by means of switches, inspection machines, controllers, and computers, respectively. A specific example is the Statistical Quality Control $\bar{X}$ and σ computer, which obviates manual chart plots and calculations.

However, we feel that mere mechanization of obvious routines (such as SQC) is not sufficient. We advocate less passive measures, such as the active application of the "scientific method," for use in the production plant.

The "scientific method" (oversimplified no doubt) consists of providing general solutions (in the form of equations) to a recurring or continuous or continual "process" by proper isolation, evaluation, and analysis of the process constants, parameters, and variables. This approach—writing equations of a seemingly unpredictable process—generalizes the process. This is equivalent to routinizing the process, because the equation can be solved in a routine manner after the values of the factors are merely "plugged-in." Thus a varying process can be controlled automatically by means of a *mentor* (control computer) that continually reconciles the process factors—constants, parameters, and variables—usually by solving a battery of process simultaneous equations.

The procedure of rendering a seemingly arbitrary process subject to control in a routine manner—by writing descriptive equations and then bringing about control by continuous automatic solution of the equations—constitutes a high order of *mechanization of information.* It is comparable to high order *mechanization of energy*, as exhibited by Detroit brand automation.

Now the only limit to the mechanization of information by means of control computers is the mathematics involved. What physical or industrial process, including mental operations, cannot be expressed in mathematical form? The only real problem is in locating the problem—that is, in finding a potential area for computer control. Then the applied mathematician will seldom be unable to express the process in mathematical form. Following this, the system control engineers can shape up the computer hardware largely from off-the-shelf components.

Because of the interest of the press in "giant brains," computer control may bring forth visions of large differential analyzers or digital computers. True, such large *general purpose* analog and digital computers can be and are useful for control purposes. However, it is *special purpose* computers that are more likely to be used for control purposes. Furthermore, *equation solvers* rather than *computers* (in a technical sense) are used for control. Such *equation solvers* (as compared to *differential analyzers*) are usually "special purpose, electromechanical, servotype, analog equation solvers." This is why we prefer the short term *mentors*. Special purpose digital computers too are used for control purposes.

We learned our computer control techniques via the military training aids route (flight trainers, gunnery trainers, navigation trainers, etc.). But, we feel that computer control need not be confined to simulation, the creation of "synthetic training situations." We claim that computer control can provide valuable service on the industrial

* Received by the PGAC, March 22, 1958.

scene in mechanizing routine mental operations used for control purposes.

Our biggest problem so far has not been the engineering involved, but abstracting the mathematical formulas defining process operation. As mathematical amateurs, we found that Polya's "How to Solve It"[1] gave us some help in "setting up equations." More directly applicable to our problem of writing equations descriptive of industrial operations was Blackett's classic paper.[2] Professor Blackett's discussion of "the variational method" (as opposed to the *a priori* method) for finding a general solution seems to us eminently suitable for application in the industrial plant. Not that we have thus far followed up this method to any appreciable extent. At best our mathematical excursions would not be adequate. We could see, as already pointed out, that the practical approach to computer control is that the process control engineers and mathematicians both be members of the control system design team and meet on the common ground of operations research. In fact we view computer control as a specialty falling into the broader discipline of Operations Research.

The writers appreciate the potentials of OR, and believe that it is not living up to its potential. One reason is that OR seems to be considered suitable mainly for advising high-level executive decisions. A practically unlimited area for OR consists of writing equations of everyday industrial processes to make them amenable to computer control. Needless to say, not until machine builders have been educated to design their machines and manufacturing processes on an OR basis will computer control be commonplace.

Before this great day comes, the advocates for computer control (such as the writers) must familiarize themselves with OR—not that they will ever be on a par with OR professionals. Conversely, OR practitioners should familiarize themselves with the arts of computer control—that is, OR people should know the functional aspects of what computer control can do. The operational aspects and hardware of *how* the computation control is accomplished can be left to the control engineers.

This closes our case in behalf of "Operations Research for Computer Control."

G. H. Amber
P. S. Amber
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19925 Schaefer Hwy.
Detroit 35, Mich

[1] G. Polya, "How to Solve It," Doubleday & Co. (Anchor Books), New York, N. Y., 2nd ed.; 1957.

[2] P. M. S. Blackett, "Operations research," *Adv. Science*, vol. 5; April, 1948. Obtainable from Edmund C. Berkeley and Assoc., 36 W. 11 St., New York 4, N. Y.

PGAC News

CURRENT REVIEWERS

The following members have reviewed one or more papers for the PGAC during the past year. Actually there are many more PGAC reviewers listed in the records, but many of them have been inactive due to other commitments in the past year. Competent, active reviewers are always needed, and, as in the past, interested persons are urged to contact the editor, indicating their willingness to participate and their special field of interest. The efforts of the active reviewers are greatly appreciated.

M. R. Aaron	W. M. Gaines	D. P. Lindorff	M. G. Rekoff, Jr.
E. Arthurs	J. E. Gibson	M. Loebel	A. S. Robinson
G. A. Biernson	A. J. Groszer	M. V. Mathews	P. M. Schultheiss
J. F. Buchan	V. B. Haas	M. McLarin	L. Simon
N. H. Choksy	R. S. Hoffman	J. H. Mulligan, Jr.	D. J. Simmons
F. F. Cilyo	R. E. Horn	G. J. Murphy	P. G. Spink
R. Clay	R. E. Kalman	J. A. Narud	T. M. Stout
H. Eckhardt	J. B. Kreer	W. A. Ostaff	F. B. Tuteur
R. L. Edwards, Jr.	J. Kukel	R. H. Plath	L. A. Zadeh
H. Freeman			F. Zweig

Dallas Conference[1]

The PGAC will sponsor a National Automatic Control Conference on November 4–6, 1959, in Dallas, Tex., at the new Sheraton-Dallas Hotel. Control groups from other organizations such as the PGIE, AIEE, ASME, and ISA, will participate in the activities.

Although the deadline for papers is not until June 1, 1959, four copies of summaries should be submitted as soon as possible to G. S. Axelby, Westinghouse Electric Corporation, Box 746, Baltimore 3, Md.

In order to facilitate selection of papers, the 1000 to 1500-word summary *must:*

1) State clearly what has been accomplished.
2) Indicate whether (a) the material is primarily theoretical or experimental, (b) practical applications are included, and (c) the paper is believed to be an original contribution or an extension of an earlier paper.
3) Include a pertinent bibliography.

Accepted papers will be published in these Transactions.

[1] Papers submitted for the Moscow Conference, announced below, may be submitted to the Dallas Conference also. Although the Moscow paper abstracts are due first, they will be presented after the Dallas Conference.

Moscow Conference[2]

The first International Congress of the International Federation of Automatic Control (IFAC) will be held in Moscow in 1960. Details of the conference as recorded in the first draft of the scientific program are given in the following sections.

Anyone desiring to present a paper at the Moscow Congress is requested to prepare an abstract and a rough outline of the proposed paper and submit them by March 1, 1959 as noted below.

[2] This material was furnished by Harold Chestnut, first President of IFAC.

General Characteristics

1) The First International Congress of IFAC will take place according to the resolution of the General Assembly of IFAC, adopted in Paris, September 12, 1957.
2) The Congress is deemed to cover three kinds of activity of scientists and engineers in the field of automatics:

   Section I—Theory and methods of automatic control and servomechanisms.

   Section II—Instrumentation and the problems connected with its constructing on new scientific base.

   Section III—Industrial applications of automatic control (including the application of prediction and computing devices).

   The Congress should pass under the motto "Wide applications—to the theory, maximum reliability—to the instrumentation, high efficiency—to the process control." The Congress will last 10 days and contain:

   a) Reading of scientific reports and their discussion.
   b) Excursions to scientific institutions and also to different automatic undertakings.
   c) Attendance at the culture centers of the Soviet Union. It is supposed to make a special program of excursions and attendings of the culture centers of Moscow available for ladies.

Scientific Program

According to the three kinds of activity of scientists and engineers in the field of automatics which are noted above, the following scientific program of the Congress is supposed (this program is described in three tables):

In the first section the general and particular theoretical problems of research and design of automatic control systems and the methods of their solution are enumerated.

Any general or particular mathematical aspect of any problem and (precise or approximate) method of its solution is admitted. The range of research in the field of theory includes mathematical problems, the aim of which is to develop new methods of their solution, as well as concrete physical problems, the aim of which is the receiving of particular results of scientific and practical meaning and their use in engineering designs.

The subjects of research can be closed-loop or open-loop systems with some inputs and outputs with constant or variable parameters, determined or stochastic, and also systems containing a man or computers as links.

In the second section, the subjects of research first of all are elementary components of instrumentation: measuring elements, transducers, transformers, relays, regulators, final control elements, etc. The general problem of the theory and design is the determining of the accuracy of present elements and also their static and dynamic characteristics.

This problem refers to any instrumentation system.

In the same section the most important problem of instrumentation—the problem of its maximum reliability—will be considered. The solution of this problem can be illustrated with the results of experimental study and application of instrumentation. The description of the principles of instrumentation design, the experiences of design and production, and the problem of economics are also discussed. New physical principles are introduced which can be incorporated in the development and manufacture of new devices, automatic systems, control and computing devices.

In the third section, methods of theoretical and experimental study of static and dynamic analysis of objects and processes, and methods of analysis of disturbances are considered. All these data—as well as mathematical interpretation of automatized objects and processes and the applications of these objects—will be used for the development of scientific principles of automation. These form a basis for consideration of the principles of construction of large control systems and automatic units, and the principles of automatic process control. Finally, the problems of improving conditions and productivity and of making automation economically efficient are considered.

In all sections the problems of history of automatics, and of international terminology in the field of automatic control are foreseen.

Procedure of Preparation and Selection of Reports

The following procedure of selecting and preparing of reports for the Congress is anticipated:

1) In every country the selecting of the reports for the Congress must be made by the National member (National Committee, AACC in the U. S.).
2) The report must contain some scientific (theoretical or experimental) or practical results of the work of one or a group of the authors in any of the enumerated fields, or in a new field, applied to the topics, dealing with development of the methods of automatic systems design or their applications in industry.
3) The reports selected by national members (National Committee) for presentation at the Congress must be presented to the Executive Council of IFAC.
4) All reports presented by the national members (National Committee) for presentation at the Congress, and approved by the Executive Council of IFAC, will be published in the Proceedings of the Congress in English and in Russian.
5) Individual scientists and engineers from every country can take part in the Congress and can take the floor without representation by their national members. However, there must be a special decision of the Executive Council about their reports.
6) The reports will be presented in Russian and English.
7) It is desirable that the size of the reports should be not more than 6000 words.

Preparation of the Congress

1) The Congress will begin on June 25, 1960. The duration of the Congress is 10 days.
2) An abstract, rough draft, or paper, must be submitted to Dr. Eugene M. Grabbe, Chairman of Moscow Congress Technical Program, The Ramo-Wooldridge Corporation, Rm. 2208, Building C, P. O. Box 45067, Airport Station, Los Angeles 45, Calif., by March 1, 1959.
3) The AACC is to notify authors, and send the title, author's name, and abstract to IFAC by May 1, 1959.
4) Complete papers must be sent to the National AACC Committee by July 15, 1959.
5) Complete papers must be sent to IFAC with recommendations for oral presentation, September 15, 1959.
6) The preparation of the reports for publishing, and agreement with authors will be made by December 31, 1959.
7) The preprint publishing will be done by March 31, 1960, in Russian and English.
8) The distribution of the reports to the authors and the participants of the Congress will be made by April 31, 1960.
9) The Congress will take place at Moscow State University. The first day of the Congress is devoted to plenary meeting. At this meeting will be:
   a) The opening of the Congress.
   b) A report, "Scientific principles of automatic control and the technical process."
10) The Sections will work from the second to ninth days inclusive.
11) The last day will feature the plenary meeting, with reports of the section leaders and current affairs of the Federation. The resolution will be taken, and different speeches will take place.
12) Sections will be created after the reports are received; sections will be presented to the Congress and will depend on the contents of the reports.

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IRE TRANSACTIONS

ON

AUTOMATIC CONTROL

1956–1958

IRE Transactions on Automatic Control

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